

UBC Physics 102 Section 951

Midterm Test 3

July 21, 2003

Instructions:

1. Do not open this test until told to do so.
2. This test is closed book. You may NOT bring any material in with you. Calculators are allowed. A formula sheet is provided on the last page of this test.
3. Print your name and student number on ALL pages.
4. Show all your work to justify your answers.
5. Use the back of the page if you need more space.
6. If you use pencil there will be NO changes to your mark once your paper has been handed back.

Marks:

	Q1	Q2	Q3	Q4	TOTAL
Mark					
Max	10	10	10	10	40

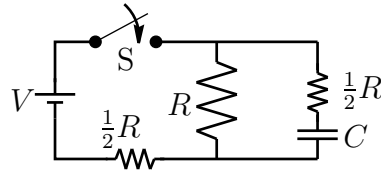
Last Name: _____ **Solution**

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Student Number: _____ **00000000**

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1. In the circuit shown, switch S is closed at time $t = 0$.



(a) (3 points) What is the current I leaving the battery at $t = 0$, immediately after the switch is closed?

Answer: Initially the capacitor is uncharged so the current passes through it as if it was a short-circuit. So the two resistors on the right act in parallel,

$$R_1 = \left[\frac{1}{R} + \frac{2}{R} \right]^{-1} = \frac{R}{3}. \textcircled{1}$$

This equivalent resistance is in series with the bottom resistor,

$$R_2 = R_1 + \frac{R}{2} = \frac{R}{3} + \frac{R}{2} = \frac{5R}{6}. \textcircled{1}$$

From Ohm's law, the current through the battery is

$$I = \frac{V}{R_2} = \frac{6V}{5R}. \textcircled{1}$$

(b) (2 points) What is the current I a "long time" later?

Answer: After a long time the current through the capacitor will fall off to zero $\textcircled{1}$ so all the current goes through the near right and bottom resistors in series,

$$R_{eq} = R + \frac{1}{2}R = \frac{3}{2}R. \textcircled{1}$$

Again, from Ohm's law

$$I = \frac{V}{R_{eq}} = \frac{2V}{3R}.$$

- (c) (2 points) What charge has accumulated on the capacitor after this long time?

Answer: There is no voltage drop across the far right resistor (zero current) so the voltage drop across the capacitor is the same as that across the near right resistor R ,

$$V_C = V_R = IR = \frac{2}{3}V. \textcircled{1}$$

The charge on a capacitor is given by

$$Q = CV_C = \frac{2}{3}CV. \textcircled{1}$$

- (d) (3 points) Finally, switch S is opened again. How long will it take after the switch is opened for the capacitor to lose 80 percent of its charge if $R = 20 \text{ k}\Omega$ and $C = 1.0 \text{ }\mu\text{F}$?

Answer: When the switch is opened the only circuit is through the capacitor and the two resistors on the right. Their equivalent resistance is

$$R_{eq} = R + \frac{1}{2}R = \frac{3}{2}R = 30 \text{ k}\Omega. \textcircled{1}$$

For a simple RC circuit the time constant is

$$\tau = R_{eq}C = (30 \text{ k}\Omega)(1.0 \text{ }\mu\text{F}) = 30 \text{ ms}. \textcircled{1}$$

Eventually the charge on the capacitor falls to zero, $Q_\infty = 0$. Because $Q = CV$ the charge declines exponentially according to

$$\begin{aligned} Q - Q_\infty &= (Q_0 - Q_\infty)e^{-t/\tau} \\ Q &= Q_0e^{-t/\tau}. \end{aligned}$$

Isolating t gives

$$t = -\tau \ln \frac{Q}{Q_0} \textcircled{1} = -(30 \text{ ms}) \ln(0.20) = 48 \text{ ms}.$$

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2. A beam of electrons (initially at rest) is accelerated to the right through 25 kV. (Electron: mass = 9.11×10^{-31} kg, charge = -1.60×10^{-19} C.)
- (a) (2 points) What is the final speed of the electrons in the beam?

Answer: The gain in kinetic energy is $K = qV$ ①. Equating that to $K = \frac{1}{2}mv^2$ we get

$$\begin{aligned} v &= \sqrt{\frac{2K}{m}} = \sqrt{\frac{2qV}{m}} \text{ ①} \\ &= \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(25 \text{ kV})}{(9.11 \times 10^{-31} \text{ kg})}} \\ &= 9.4 \times 10^7 \text{ m/s.} \end{aligned}$$

- (b) (3 points) What value of magnetic field would make the beam, travelling to the East at speed 1.0×10^8 m/s, go undeflected through a region where there is a uniform electric field of 20 kN/C pointing vertically up? (Neglect gravity.)

Answer: To prevent deflection the magnetic force must exactly cancel the electric force. The electric force is given by $F_E = qE$ ①, The magnetic force on a moving particle is $F_B = qvB$ ①. To balance $F_B = F_E$ we need a magnetic field of

$$\begin{aligned} B &= \frac{F_E}{qv} = \frac{E}{v} \text{ ①} \\ &= \frac{20 \times 10^3 \text{ N/C}}{1.0 \times 10^8 \text{ m/s}} \\ &= 2.0 \times 10^{-4} \text{ T.} \end{aligned}$$

- (c) (2 points) What is the direction of the magnetic field?

Answer: The electric field is upwards, acting on negatively charged electrons, forcing them down. So the magnetic force has to be upwards. ①

The right-hand force rule tells us that an eastbound *positive* charge will feel an upwards force if the B -field is northwards but these are *negative* charges so the B -field has to point south. ①

Out of page would also be an acceptable answer if you defined “vertically up” as up the page and East to the right.

- (d) (3 points) The electric field is turned off. What is the period of one circular orbit of the electrons if the magnetic field strength is 1.0×10^{-4} T? (Hint: $a = \frac{v^2}{r}$ in a circular orbit.)

Answer: With only the magnetic force acting on the electrons they will start moving in a circle because the force is always perpendicular to the motion. We are told $a = \frac{F}{m} = \frac{v^2}{r}$ ① so the orbit radius is

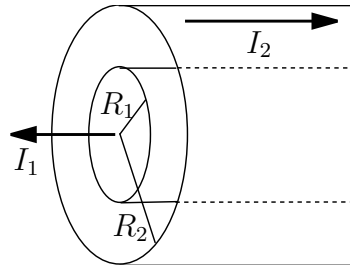
$$r = \frac{mv^2}{F_B} = \frac{mv^2}{qvB} = \frac{mv}{qB} \cdot ①$$

The period T is the time it takes to travel one circumference,

$$\begin{aligned} T &= \frac{2\pi r}{v} = \frac{2\pi m}{qB} \cdot ① \\ &= \frac{2\pi(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(1.0 \times 10^{-4} \text{ T})} \\ &= 3.6 \times 10^{-7} \text{ s.} \end{aligned}$$

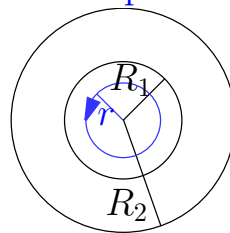
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3. A long coaxial cable consists of a solid inner conducting core of radius R_1 , surrounded by a concentric cylindrical sheath of radius R_2 , as shown. The core carries a uniformly distributed current I_1 , and a larger current $I_2 > I_1$ is carried in the opposite direction in the sheath.



- (a) (3 points) Determine the magnetic field in the core, at a distance r from the axis ($r < R_1$).

Answer: The symmetry of the problem suggests Ampère's law would be a good choice. Let's draw a loop at radius r inside the core.



From the RH field rule and symmetry the B -field must be parallel to the loop everywhere so $B_{\parallel} = B$ and the length of the loop is $l = 2\pi r$. All we need now is the enclosed current. The current is carried uniformly over the area so

$$I_{\text{encl}} = I_1 \frac{r^2}{R_1^2} \cdot \textcircled{1}$$

So Ampère's law becomes

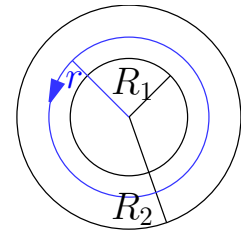
$$\begin{aligned} B_{\parallel} l &= \mu_0 I_{\text{encl}} \cdot \textcircled{1} \\ B(2\pi r) &= \mu_0 I_1 \frac{r^2}{R_1^2} \\ B &= \frac{\mu_0 I_1}{2\pi R_1^2} r \cdot \textcircled{1} \end{aligned}$$

- (b) (2 points) Determine the magnetic field at a distance r from the axis for $R_1 < r < R_2$.

Answer:

Again, we use Ampère's law, now with a circle in the gap between the conductors. The B -field is still $B_{\parallel} = B$ and the length is still $l = 2\pi r$.

The only thing that's changed is that the enclosed current is now constant $I_{\text{encl}} = I_1$ ① so



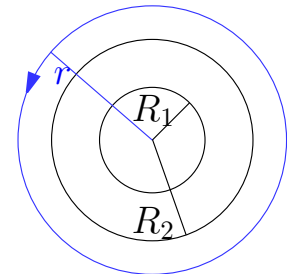
$$B(2\pi r) = \mu_0 I_1$$

$$B = \frac{\mu_0 I_1}{2\pi r} \text{ ①}$$

- (c) (2 points) Determine the magnetic field at a distance r from the axis for $r > R_2$.

Answer:

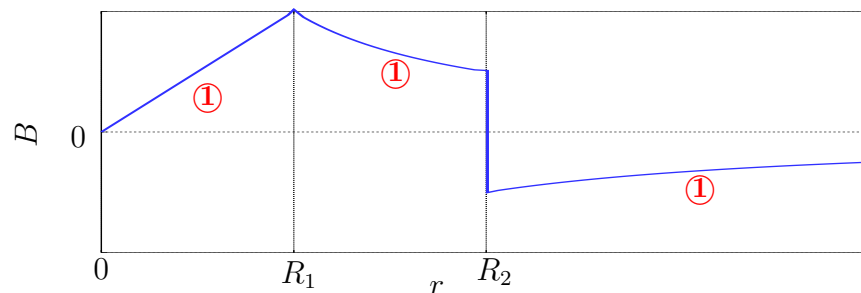
The trick this time is to consider the direction of the current. Let's take the path going around in the same direction as before. Then, by the RH field rule, the current producing a B -field in that direction is $I_{\text{encl}} = I_1 - I_2$ ①. From Ampère's law we get



$$B = \frac{\mu_0 (I_1 - I_2)}{2\pi r} \text{ ①}$$

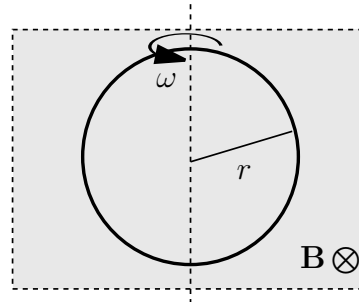
Notice that since $I_1 < I_2$, B will be negative, meaning it is oriented *antiparallel* to our path.

- (d) (3 points) Sketch the magnetic field as a function of distance for all three regions, on the graph provided.



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4. A circular loop of copper wire of radius r is rotated at angular speed ω in a uniform magnetic field of strength B , as shown.



- (a) (3 points) What is the emf generated in the wire as a function of time t (in terms of r , B , and ω)?

Answer: Faraday's law states that the induced emf is $\mathcal{E} = -\frac{d\Phi_B}{dt}$. The flux at any moment is $\Phi_B = B_{\perp}A$. The area is just $A = \pi r^2$ but the field is changing relative to the area as

$$B_{\perp} = B \sin \omega t.$$

So the emf is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = A \frac{dB_{\perp}}{dt} = \pi r^2 B \omega \cos \omega t.$$

(Or you could have written $\mathcal{E} = (\dots) \sin \omega t$ because it's just a matter of choosing the orientation at $t = 0$.)

- (b) (2 points) If the wire has a circular cross-section with a diameter of 4.0 mm and has resistivity $1.7 \times 10^{-8} \Omega \cdot \text{m}$ what is the resistance of the loop, for a loop radius of 10 cm?

Answer: The equation for resistance is $R = \rho \frac{l}{A}$. The length of the loop is $l = 2\pi(10 \text{ cm}) = 0.62 \text{ m}$ and the cross-sectional area is $A = \pi r^2 = \pi(2.0 \text{ mm})^2 = 1.3 \times 10^{-5} \text{ m}^2$ so the resistance works out to

$$R = \rho \frac{l}{A} = (1.7 \times 10^{-8} \Omega \cdot \text{m}) \frac{0.62 \text{ m}}{1.3 \times 10^{-5} \text{ m}^2} = 8.4 \times 10^{-4} \Omega.$$

- (c) (1 point) Assume the resistance of the loop is R . What is the current in the wire as a function of time?

Answer: The emf induces a current given by Ohm's law, $\mathcal{E} = IR$ ①
so the current is

$$I = \frac{\mathcal{E}}{R} = \frac{\pi r^2 B \omega}{R} \cos \omega t.$$

- (d) (4 points) How much work is required to perform one complete rotation of the loop?

Answer: The work that is put into turning the rotation has to be "used up" somewhere (conservation of energy); in this case it gets turned into heat via the resistance of the loop itself. We can find the average power lost through the resistor by recognizing that the voltage and current are simply sinusoidal so the average power lost is $\overline{P} = V_{RMS} I_{RMS}$ ①. The RMS values are

$$V_{RMS} = \frac{V_0}{\sqrt{2}} = \frac{\pi r^2 B \omega}{\sqrt{2}}$$

and

$$I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{\pi r^2 B \omega}{\sqrt{2} R}.$$

So the average power drain is

$$\overline{P} = V_{RMS} I_{RMS} = \frac{\pi^2 r^4 B^2 \omega^2}{2R}. \text{ ①}$$

The energy consumed in one oscillation, $\omega t = 2\pi$, ① is

$$W = \overline{P} t = \frac{\pi^2 r^4 B^2 \omega^2}{2R} \frac{2\pi}{\omega} = \frac{\pi^3 r^4 B^2 \omega}{R}. \text{ ①}$$

END OF EXAM

(Did you print your name and student number on every page?)