Tutorial 2 Question

- Text: Ch. 42: Pr. 54.
- An ancient club is found that contains $190\ \mathrm{g}$ of carbon and has an activity of 5.0 decays per second. Determine its age assuming that in living trees the ratio of $^{14}\mathrm{C}/^{12}\mathrm{C}$ atoms is about 1.3×10^{-12}.

Solution

- We know $\boxed{N(t) = N_0 e^{-\lambda t}}$. Want to solve for t, so

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N}.$$

- Need to find λ, N_0, and N.
- First λ. Given $\boxed{T_{1/2} = \frac{\ln 2}{\lambda}}$ we find

$$\begin{aligned} \lambda &= \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5730 \text{ yr}} \\ &= 1.21 \times 10^{-4} \text{ yr}^{-1} \\ &= 3.83 \times 10^{-12} \text{ s}^{-1}. \end{aligned}$$

(Note: $1 \text{ yr} \approx 3.166 \times 10^{7} \text{ s}$.)

Solution, contd

- Now N_0. Initially, a fraction 1.3×10^{-12} of the carbon was $^{14}\mathrm{C}$.
- Using that and $\boxed{1\ \mathrm{u} = 1.66 \times 10^{-27}\ \mathrm{kg}}$ we can calculate the initial number of $^{14}\mathrm{C}$ atoms,

$$\begin{aligned} N_0 &= (1.3 \times 10^{-12})(0.190\ \mathrm{kg}) \times \frac{1\ \mathrm{u}}{1.66 \times 10^{-27}\ \mathrm{kg}} \times \frac{1\ \mathrm{atom}}{12\ \mathrm{u}} \\ &= 1.2 \times 10^{13}\ \mathrm{atoms}. \end{aligned}$$

- We used an atomic mass of $12\ \mathrm{u}$ because almost all of the carbon is $^{12}\mathrm{C}$.
- Lastly N. We are given the activity $\left|\frac{dN}{dt}\right| = 5.0\ \mathrm{atoms/s}$.

Solution, contd

- Differentiating $\boxed{N(t) = N_0 e^{-\lambda t}}$ gives

$$\begin{aligned} \frac{dN}{dt} &= -\lambda N_0 e^{-\lambda t} \\ &= -\lambda N. \end{aligned}$$

- So we find

$$\begin{aligned} N &= \frac{1}{\lambda}\left|\frac{dN}{dt}\right| \\ &= \frac{1}{3.83 \times 10^{-12}\ \mathrm{s}^{-1}} \times 5.0\ \mathrm{atoms/s} \\ &= 1.3 \times 10^{12}\ \mathrm{atoms}. \end{aligned}$$

Solution, contd

- That's all the information we need to solve for t,

$$
\begin{aligned}
t &= \frac{1}{\lambda} \ln \frac{N_0}{N} \\
&= \frac{1}{3.83 \times 10^{-12}\ \mathrm{s}^{-1}} \ln \left(\frac{1.2 \times 10^{13}\ \mathrm{atoms}}{1.3 \times 10^{12}\ \mathrm{atoms}} \right) \\
&= 5.8 \times 10^{11}\ \mathrm{s} \\
&= 18,000\ \mathrm{yr}.
\end{aligned}
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- The club is around 18,000 years old. □