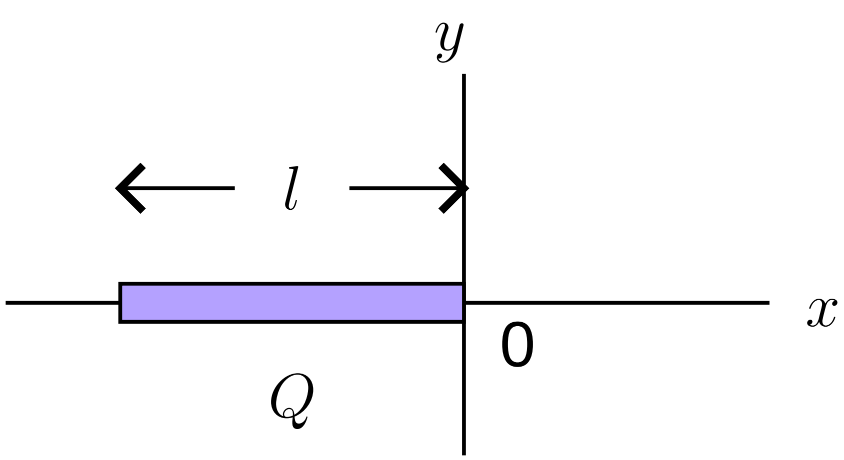
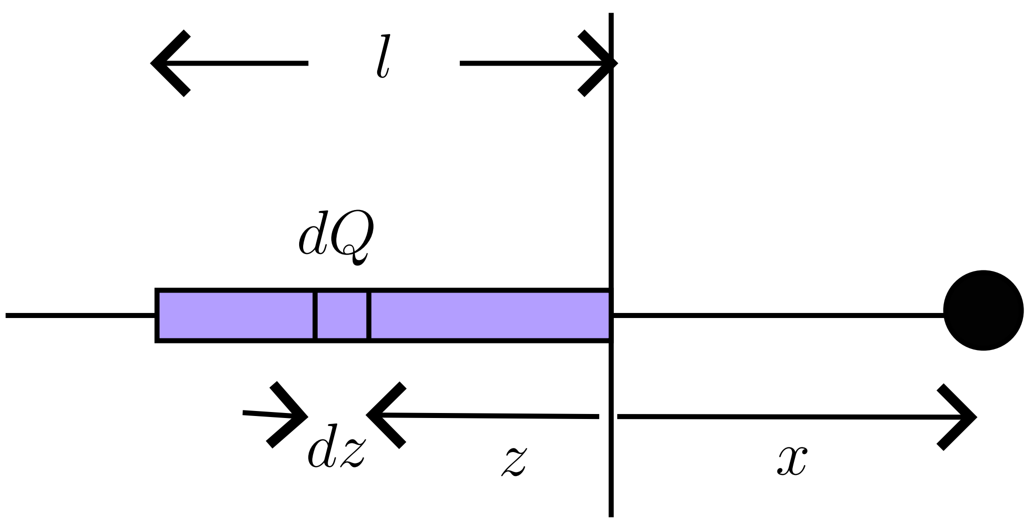
Tutorial 5 Question

- Text: Ch. 21: Pr. 48.
- A thin rod of length l carries a total charge Q distributed uniformly along its length. Determine the electric field along the axis of the rod starting at one end—that is, find $E(x)$ for $x \geq 0$.
- Hint: $\int \frac{dz}{(x+z)^2} = \frac{-1}{x+z} + C$.



Solution

- Since this is a continuous charge distribution we have to break it into small chunks and add all the electric fields together.
- To break the rod into chunks we introduce a new variable z which indicates our current position along the length of the rod. Then we look at a chunk of rod of length dz.



Solution, contd

- The charge in that chunk is $dQ = \frac{Q\,dz}{l}$. And it is at a distance $r = x + z$ from the point x so, from Coulomb's law the electric field is

$$\begin{aligned} d\mathbf{E} &= \frac{k\,dQ}{r^2}\hat{\mathbf{r}} \\ &= \frac{kQ\,dz}{l(x+z)^2}\hat{\mathbf{i}}. \end{aligned}$$

- That gives the contribution from the chunk at position z. To get the total field we add up the contributions for all z between 0 and l to get

$$\mathbf{E} = \int d\mathbf{E}$$

Solution, contd

$$
\begin{aligned}
\mathbf{E} &= \int_0^l \frac{kQ\,dz}{l(x+z)^2}\hat{\mathbf{i}} \\
&= \frac{kQ}{l}\hat{\mathbf{i}} \int_0^l \frac{dz}{(x+z)^2} \\
&= \frac{kQ}{l}\hat{\mathbf{i}}\,\frac{l}{(x+l)x} \\
&= \frac{kQ}{(x+l)x}\hat{\mathbf{i}}. \quad \square
\end{aligned}
$$

- Nontrivial integrals will be given if needed on tests.
- The question only asks for the scalar field strength. I included the vector to demonstrate how it can be worked through the integral.