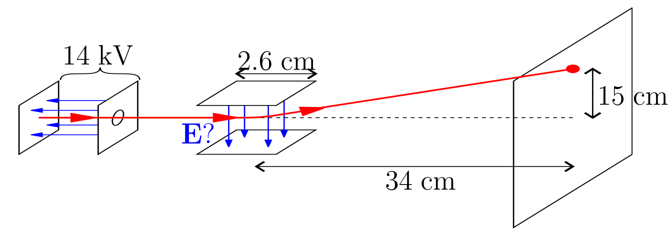
Tutorial 7 Question

- Text: Ch. 23: Pr. 68.
- Electrons are accelerated by $14\ \mathrm{kV}$ in a CRT. The screen is $30\ \mathrm{cm}$ wide and is $34\ \mathrm{cm}$ from the $2.6\ \mathrm{cm}$-long deflection plates. Over what range must the horizontally deflecting electric field vary to sweep the beam fully across the screen?

Solution



- The question is asking what the strength of the electric field across the deflecting plates must be to achieve a deflection that just hits the edge of the screen.
- We can solve this by finding the speed in the y and z directions an electron has after it has passed through the deflector.

Solution, contd

- The first step is to work out the speed in the z direction, v_z. The electron is accelerated through $14\ \mathrm{kV}$ so it gains kinetic energy

$$\begin{aligned}\Delta K &= -\Delta U = -qV \\ &= (1.60 \times 10^{-19}\ \mathrm{C})(14\ \mathrm{kV}) \\ &= 2.24 \times 10^{-15}\ \mathrm{J}.\end{aligned}$$

- If we assume the electron started at rest then ΔK is its kinetic energy after acceleration so, from $K = \frac{1}{2}mv^2$,

$$\begin{aligned}v_z &= \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.24 \times 10^{-15}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}} \\ &= 7.01 \times 10^{7}\ \mathrm{m/s}.\end{aligned}$$

Solution, contd

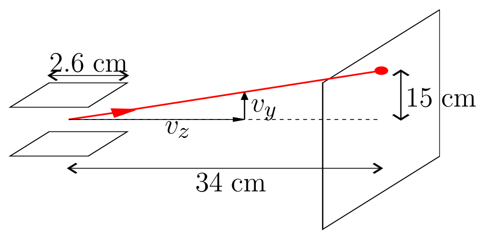
- Next, the electron will get accelerated by the E-field of the deflector.
- This time we should work with force instead of energy because we want to find E.
- Let's just consider the maximum deflection. The force applied to the electron (neglecting direction) is $F = qE = eE$.
- So the electron's y-acceleration is $a_y = F/m$ or

$$a_y = \frac{eE}{m}.$$

- So if we can find what a_y needs to be we'll be able to solve for E.

Solution, contd

- The acceleration needs to be enough so that when the electron leaves the deflector it has enough y-speed to reach the edge of the screen.



- From the diagram we see that the speed and displacement triangles are similar so, after deflection,

$$\frac{v_y}{v_z} = \frac{15 \text{ cm}}{34 \text{ cm}}.$$

Solution, contd

- So the y-speed needs to be

$$\begin{aligned} v_y &= \frac{15}{34}(7.01 \times 10^7 \text{ m/s}) \\ &= 3.09 \times 10^7 \text{ m/s}. \end{aligned}$$

- If the electron is in the deflector for some time t then its final y-speed is related to its acceleration by $v_y = a_y t$.
- We need to find t to finally solve the problem.
- Travelling at speed v_z the electron will travel the length d of the deflector in

$$\begin{aligned} t &= \frac{d}{v_z} = \frac{2.6 \text{ cm}}{7.01 \times 10^7 \text{ m/s}} \\ &= 3.71 \times 10^{-10} \text{ s}. \end{aligned}$$

Solution, contd

- That's the last bit of information we need. Now we can work out that the y-acceleration is

$$a_y = \frac{v_y}{t} = \frac{3.09 \times 10^7 \text{ m/s}}{3.71 \times 10^{-10} \text{ s}} = 8.33 \times 10^{16} \text{ m/s}^2.$$

- And the E-field needed to produce this acceleration is (from above)

$$\begin{aligned} E &= \frac{ma_y}{e} = \frac{(9.11 \times 10^{-31} \text{ kg})(8.33 \times 10^{16} \text{ m/s}^2)}{1.60 \times 10^{-19} \text{ C}} \\ &= 474 \text{ kN/C}. \end{aligned}$$

- So the deflector field has to span $-474 \leq E \leq +474$ kN/C to sweep the beam fully across the screen. □