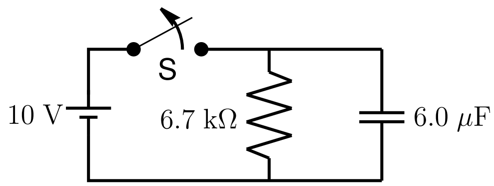
Tutorial 10 Question

- After the switch S has been closed for a long time in the circuit shown, it is opened. (*a*) What is the charge on the capacitor just after the switch is opened? (*b*) Does the current flow clockwise or counter-clockwise? (*c*) How long does it take the capacitor to discharge to $0.1\ \mathrm{V}$? (*d*) What is the voltage across the resistor at this time? (*e*) There is something missing in this circuit. What is it and why does it need to be there? (Hint: what happened when the switch was first closed?)



Solution

(*a*) What is the charge on the capacitor just after the switch is opened?

- If the switch has been closed a long time we can assume the voltage across the capacitor (or resistor) has reached $10\ \mathrm{V}$. So the charge on the plates is

$$\begin{aligned} Q &= CV = (6.0\ \mu\mathrm{F})(10\ \mathrm{V}) \\ &= 60\ \mu\mathrm{C}. \end{aligned}$$

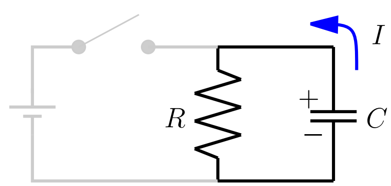
(*b*) Does the current flow clockwise or counter-clockwise?

- When the switch was closed a positive charge built up on the upper plate of the capacitor and a negative charge on the lower plate.
- When the switch is opened there is no longer a voltage holding those charges in place.

Solution, contd

(*b*) contd

- So the current flows from the upper plate to the lower along the wire—in a **counterclockwise** direction.



(*c*) How long does it take the capacitor to discharge to $0.1\ \mathrm{V}$?

- We can now ignore the left loop in the circuit because the switch is open so we are just dealing with a capacitor and resistor in series.

Solution, contd

(*c*) contd

- In that case the time constant τ is just

$$\tau = RC = (6.7\ \mathrm{k}\Omega)(6.0\ \mu\mathrm{F}) = 40\ \mathrm{ms}.$$

- In the steady state all the charge will leak off the capacitor so $V_\infty = \frac{Q_\infty}{C} = 0$. So the voltage across the capacitor follows

$$\begin{aligned} V(t) - V_\infty &= (V_0 - V_\infty)\, e^{-t/\tau} \\ V(t) - 0 &= (10\ \mathrm{V} - 0) e^{-t/(40\ \mathrm{ms})} \\ V(t) &= (10\ \mathrm{V}) e^{-t/(40\ \mathrm{ms})}. \end{aligned}$$

Solution, contd

(*c*) contd

- We just need to solve $V(t) = 0.1\ \mathrm{V}$ for t, which has the solution

$$t = -\tau \ln \frac{V(t)}{V_0}$$
$$t = -(40\ \mathrm{ms}) \ln \frac{0.1\ \mathrm{V}}{10\ \mathrm{V}}$$
$$t = 190\ \mathrm{ms}.$$

- The voltage across the capacitor reaches $0.1\ \mathrm{V}$ $190\ \mathrm{ms}$ after the switch is opened.

(*d*) What is the voltage across the resistor at this time?

- By Kirchhoff's loop rule, the voltage across the resistor must also be $0.1\ \mathrm{V}$.

Solution, contd

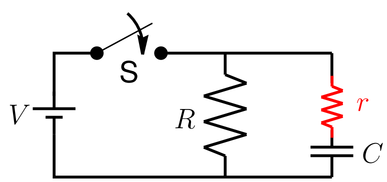
(*e*) There is something missing in this circuit. What is it and why does it need to be there? (Hint: what happened when the switch was first closed?)

- When the switch was first closed it appears the voltage across the capacitor was immediately $10\ \mathrm{V}$ (from Kirchhoff's loop rule).
- But that doesn't make sense because it takes time for a charge (and hence, voltage) to build up on the capacitor.
- So the voltage drop around that part of the circuit must have been due to something else.
- The something else could have been the resistance of the wire (which we normally take to be negligible but can't in this case).

Solution, contd

(*e*) contd

- To represent the wire resistance we should insert a resistor r in the path of the capacitor:



- As long as $r \ll R$ we can neglect it when talking about what happens after we open the switch.
- But r is important in determining how long the capacitor took to charge when the switch was first closed. □