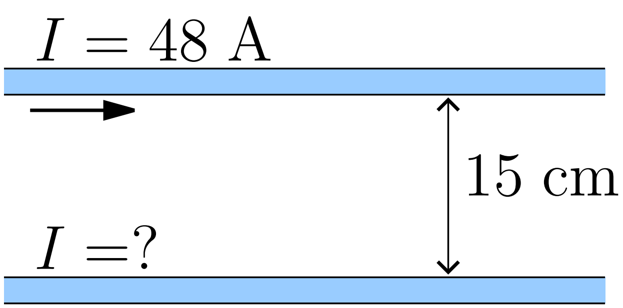
Tutorial 12 Question

- Ch 28: Pr. 52.
- A long horizontal wire carries a current of $48\ \mathrm{A}$. A second wire, made of $2.5\ \mathrm{mm}$-diameter copper wire and parallel to the first but $15\ \mathrm{cm}$ below it, is held in suspension magnetically. (*a*) What is the magnitude and direction of the current in the lower wire? (*b*) Is the lower wire in stable equilibrium? (*c*) Repeat parts (*a*) and (*b*) if the second wire is suspended $15\ \mathrm{cm}$ *above* the first due to the first's magnetic field.
- Information: Density of copper, $\rho = 8.9 \times 10^3\ \mathrm{kg/m^3}$.



Solution

(a) What is the magnitude and direction of the current in the lower wire?

- The criterion is that the magnetic force $F_m = IlB$ exactly balances gravity $F_g = mg$.
- Using both the RH-field and RH-force rules we see that that F_m will be upwards if the current on the lower wire is to the right.
- Now we need to check its magnitude. The B field due to the top wire is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(48\ \mathrm{A})}{2\pi(0.15\ \mathrm{m})}$$
$$= 6.4 \times 10^{-5}\ \mathrm{T}.$$

Solution, contd

(*a*) contd

- The magnetic force on a length l of the bottom wire (with unknown current I) is $F_m = IlB$.
- The gravitational force on the same length of wire (diameter, $d = 2.5$ mm) is $F_g = g\rho V = g\rho l(\pi d^2/4)$.
- To balance these forces we need

$$\begin{aligned} IlB &= g\rho l(\pi d^2/4) \\ I &= \frac{\pi g \rho d^2}{4B} \\ &= \frac{\pi(9.8\ \mathrm{m/s^2})(8.9 \times 10^3\ \mathrm{kg/m^3})(0.0025\ \mathrm{m})^2}{4(6.4 \times 10^{-5}\ \mathrm{T})} \\ &= 6.7 \times 10^3\ \mathrm{A}. \end{aligned}$$

Solution, contd

(*a*) contd

- So the wire will be suspended if it carries a current 6.7×10^3 A to the right. (That's *huge!*)

(*b*) Is the lower wire in stable equilibrium?

- If it's stable then if the wire slips out of position it will be pushed back into equilibrium.
- Let's think about what happens if the wire slips down a bit. Then the top wire is further away so its magnetic field is smaller. That means the magnetic force is smaller.
- So it won't be able to hold the wire up anymore and it will fall even lower.
- So this is **not** a stable equilibrium.

Solution, contd

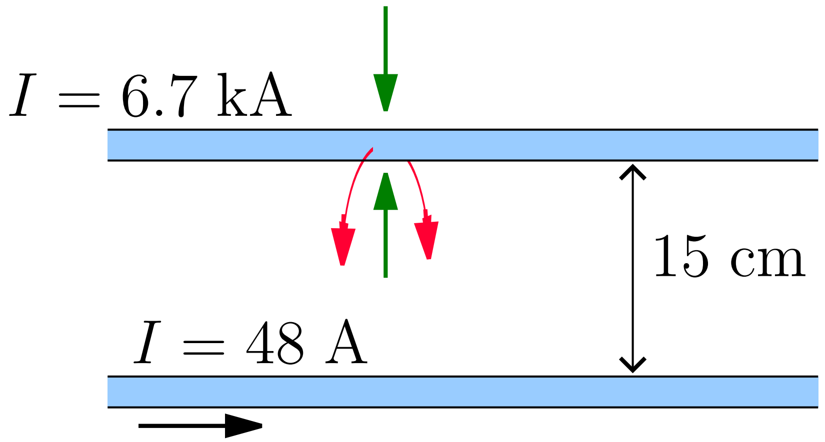
(*c*) Repeat if the second wire is suspended $15\ \mathrm{cm}$ *above* the first.

- The force has to be repulsive this time so the unknown current should run to the left.
- But the magnitude of the B-field and the force are the same, just in the opposite direction.
- So we just need to check its stability.
- This time, if the wire slips down the magnetic force pushing it back up gets *stronger* so it will move back up to equilibrium.
- So it is **stable** to displacements in the vertical direction.
- But what if it gets knocked to the side?

Solution, contd

(*c*) contd

- The magnetic force will push it radially away from the lower wire.
- Effectively, it will "slide" off the lower wire. This diagram shows the stability:



- So it won't be pulled back into equilibrium and, overall, it's still in an **unstable** equilibrium. □