Tutorial 13 Question

- Ch 29: Pr. 64.
- What is the energy dissipated as a function of time in a circular loop of ten turns of wire having a radius of $10.0\ \mathrm{cm}$ and a resistance of $2.0\ \Omega$ if the plane of the loop is perpendicular to a magnetic field given by

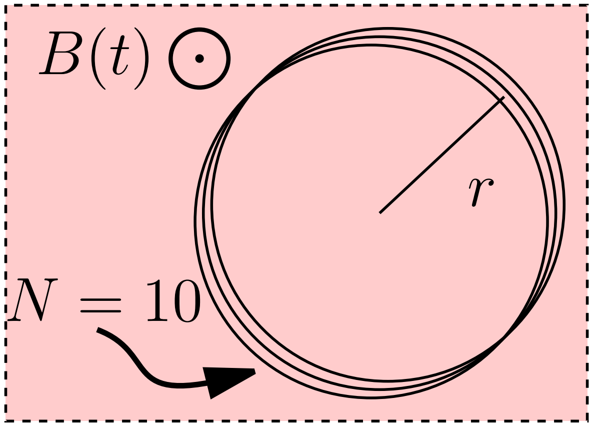
$$B(t) = B_0 e^{-t/\tau}$$

with $B_0 = 0.50\ \mathrm{T}$ and $\tau = 0.10\ \mathrm{s}$?

- Hint: $\displaystyle\int_0^T e^{-at/\tau}\, dt = \frac{\tau}{a}\left(1 - e^{-aT/\tau}\right)$.

Solution

- First, let's visualize the situation:



- The magnetic field creates a flux through the loops,

$$\Phi_B = NBA = \pi r^2 NB.$$

Solution, contd

- Since the B-field is changing so is the flux, generating an emf according to Faraday's law,

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -\pi r^2 N \frac{dB}{dt}.$$

- The B-field declines at a rate

$$\frac{dB}{dt} = \frac{d}{dt}\left(B_0 e^{-t/\tau}\right) = -\frac{B_0}{\tau} e^{-t/\tau}.$$

- Recall, we're trying to find the energy dissipated by some time T. The rate of dissipation is the power consumption of the loops, $P = I\mathscr{E}$.

Solution, contd

- So we need to know the current through the loops. From Ohm's law, $I = \frac{\mathscr{E}}{R}$ so the power consumption is

$$P = \frac{\mathscr{E}^2}{R} = \frac{\left(\pi r^2 N B_0\right)^2}{R\tau^2} e^{-2t/\tau}.$$

- If power is the rate (time derivative) of energy dissipation then the energy dissipated, E, by time T, is the integral of power,

$$\begin{aligned} E(T) &= \int_0^T P\,dt \\ &= \frac{\left(\pi r^2 N B_0\right)^2}{R\tau^2} \int_0^T e^{-2t/\tau}\,dt \end{aligned}$$

Solution, contd

$$E(T) = \frac{\left(\pi r^2 N B_0\right)^2}{R\tau^2} \frac{\tau}{2} \left(1 - e^{-2T/\tau}\right)$$
$$= \frac{\left(\pi r^2 N B_0\right)^2}{2R\tau} \left(1 - e^{-2T/\tau}\right).$$

- The last step is to just plug in the numbers given,

$$E(T) = \frac{\left(\pi (0.10\ \mathrm{m})^2 (10)(0.50\ \mathrm{T})\right)^2}{2(2.0\ \Omega)(0.10\ \mathrm{s})} \left(1 - e^{-2T/(0.10\ \mathrm{s})}\right)$$
$$= (0.062\ \mathrm{J}) \left(1 - e^{-T/(0.05\ \mathrm{s})}\right).$$

- (This tells us that after a long time the loops will have generated 0.062 J of heat.) □