Physics 153 Section T0H - Solution to Problem 1

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1 Assigned Problem

(From Tipler Ch. 31 #60.)

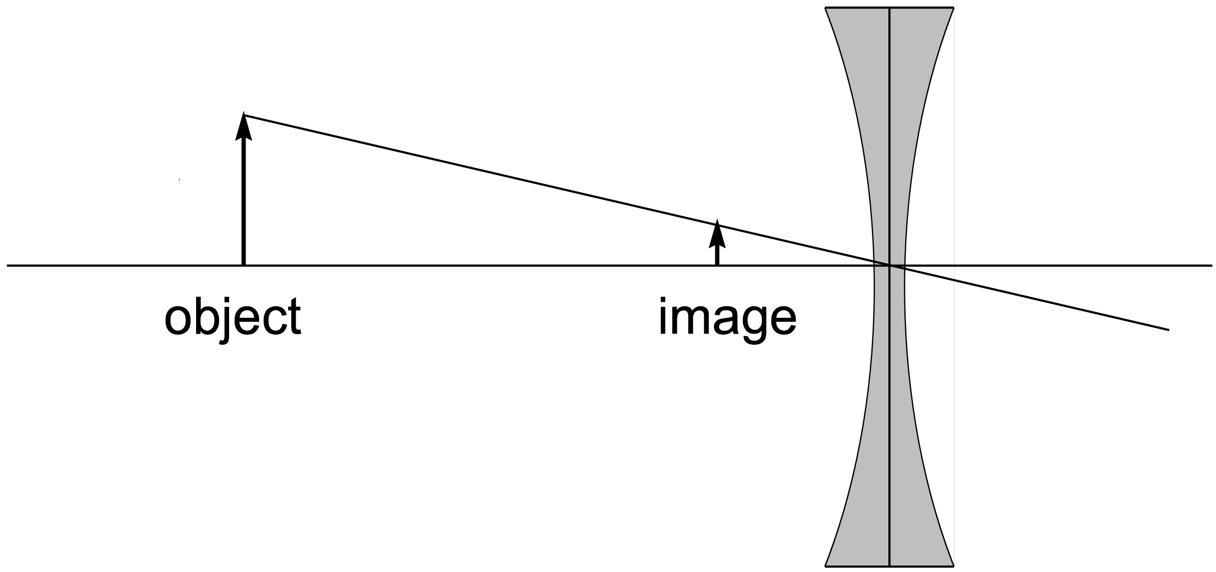
When a bright light source is placed $30\,\mathrm{cm}$ in front of a lens, there is an erect image $7.5\,\mathrm{cm}$ from the lens. *Draw a ray diagram for the lens.* There is also a faint inverted image $6\,\mathrm{cm}$ in front of the lens due to reflection from the front surface of the lens. *Draw a ray diagram for the reflection.* When the lens is turned around, this weaker, inverted image is $10\,\mathrm{cm}$ in front of the lens. *Draw a ray diagram for this reflection. Find the index of refraction of the lens.*

2 Solution

In this problem we have to consider three different images: one formed by the lens itself and two reflections created by the surfaces of the lens acting as mirrors.

2.1 Lens image

We are told that the object is at a distance $s = +30\,\mathrm{cm}$ and $s' = \pm 7.5\,\mathrm{cm}$ (we don't know which side). But we are told the image is erect so we know the image must be on the object side (because of the central ray can only produce an erect image on the object side) so $s' = -7.5\,\mathrm{cm}$.



We can also calculate the focal length of the lens:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{1}$$

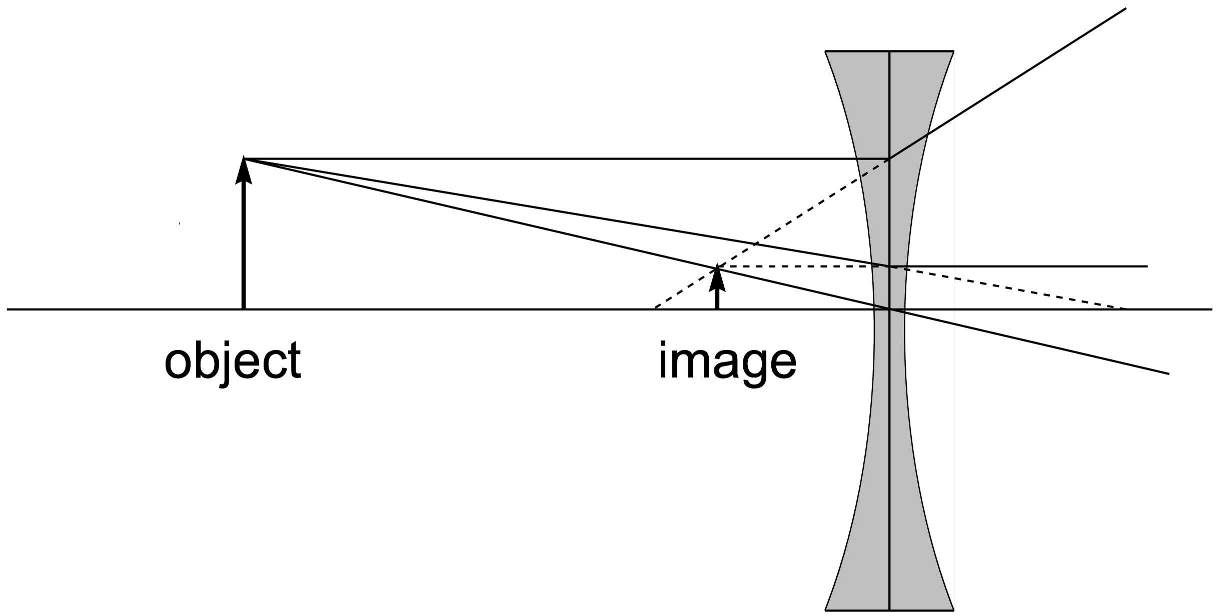
$$= \frac{1}{30} + \frac{1}{-7.5} \tag{2}$$

so

$$f_{lens} = -10\,\mathrm{cm} \tag{3}$$

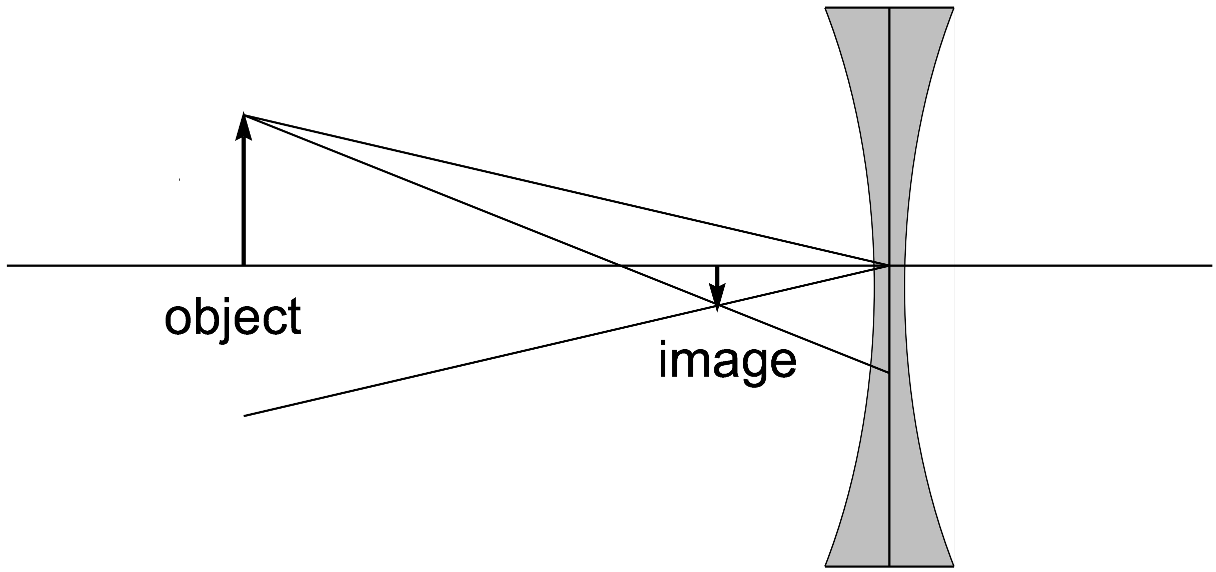
Since the focal length is negative, we know the lens is a diverging lens.

Completing the parallel ray and the focal ray gives a complete ray diagram:



2.2 First mirror image

The image produced by the reflection off the front surface of the lens is inverted so we know the lens must be concave with a ray diagram as shown:



We can also calculate the radius of curvature of the front surface from the mirror equation

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{4}$$

$$= \frac{1}{30} + \frac{1}{6} \tag{5}$$

so

$$f_1 = 5\,\mathrm{cm} \tag{6}$$

and

$$r_1 = 2f_1 = 10\,\mathrm{cm} \tag{7}$$

2.3 Second mirror image

Turning the lens around produces another reflected image and the ray diagram is pretty much the same as before.

Again, we can also calculate the radius of curvature of the back surface from the mirror equation

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{8}$$

$$= \frac{1}{30} + \frac{1}{10} \tag{9}$$

so

$$f_2 = 7.5\,\mathrm{cm} \tag{10}$$

and

$$r_2 = 2f_2 = 15\,\mathrm{cm} \tag{11}$$

2.4 Lens-maker's equation

We now know the focal length f and both radii (r_1, r_2) of the lens, which is enough to calculate the index of refraction using the lens-maker's equation. But we must be careful to remember our sign convention that r is positive if the surface is curved towards the transmission side. Hence, for this doubly-concave lens,

$$r_1 = -10\,\mathrm{cm} \tag{12}$$

$$r_2 = +15\,\mathrm{cm} \tag{13}$$

and, substituting into the lens-maker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{14}$$

$$\frac{1}{-10} = (n-1)\left(\frac{1}{-10} - \frac{1}{15}\right) \tag{15}$$

gives

$$n = 1.6. \tag{16}$$