Physics 153 Section T0H - Solution to Problem 10

Rik Blok

March 22, 2000

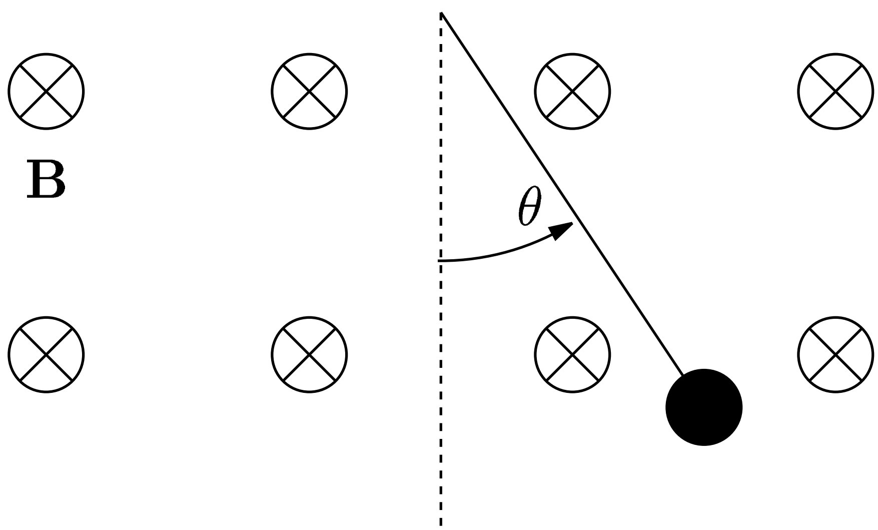
1 Assigned Problem

(From Tipler Ch. 26 #70.)

A simple pendulum has a wire of length l supporting a metal ball of mass m. The wire has negligible mass and moves in a uniform horizontal magnetic field B. This pendulum executes simple harmonic motion with angular amplitude θ_0, in a plane perpendicular to $\mathbf{B}$. What is the emf generated along the wire?

2 Solution

Ok, let's first make sure we understand what's going on. Let's take the point of view that the magnetic field is going straight into the page and the pendulum is swinging right-to-left as shown.



To find the emf we use Faraday's law which states

$$\varepsilon = -\frac{d\phi_m}{dt} \tag{1}$$

so we need to calculate the magnetic flux ϕ_m.

The pendulum is just a simple harmonic oscillator so its motion is described by the equation

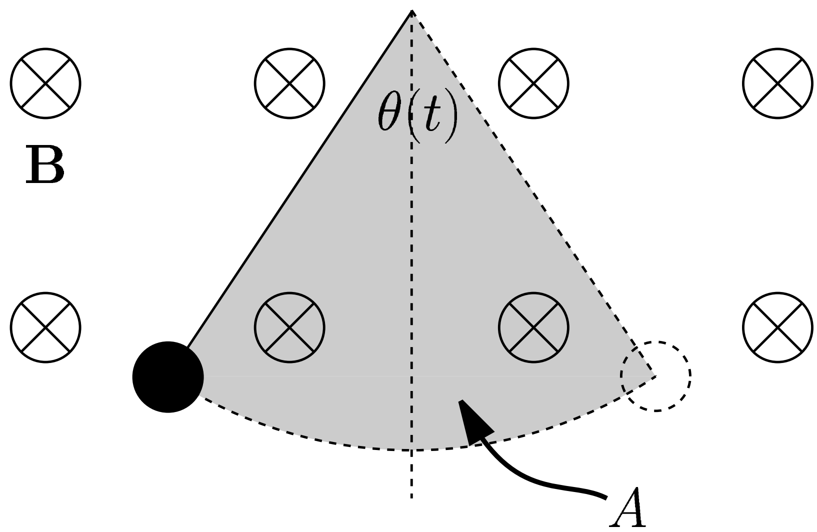
$$\theta(t) = \theta_0 \cos \omega t \tag{2}$$

(taking $\theta(0) = +\theta_0$).

The angular frequency ω is a property of the pendulum and is given by

$$\omega = \sqrt{\frac{g}{l}}. \tag{3}$$

So the pendulum sweeps a pie-shaped slice through the magnetic field with an area A, as shown below.



The total area A is some fraction of a circle. If the angle θ was a full revolution (2π) then A would just be the whole circle (πl^2) so the fraction is $\theta/2\pi$:

$$A(t) = (\pi l^2)\frac{\theta(t)}{2\pi} \tag{4}$$

$$= \frac{l^2}{2}\theta(t). \tag{5}$$

The total magnetic flux is just

$$\phi_m(t) = BA(t) = \frac{Bl^2}{2}\theta(t) \tag{6}$$

so the emf is

$$\begin{aligned}
\varepsilon &= -\frac{d\phi_m}{dt} &(7)\\
&= -\frac{Bl^2}{2}\frac{d\theta}{dt} &(8)\\
&= -\frac{Bl^2}{2}(-\theta_0\omega\sin\omega t) &(9)\\
&= \frac{Bl^2\theta_0\omega}{2}\sin\omega t &(10)
\end{aligned}$$

where $\omega = \sqrt{g/l}$.

So the swinging pendulum generates a sinusoidal alternating emf. From the right-hand-rule $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ we know the positive charges gather on the bottom when the pendulum is swinging to the right, and on the top when it comes back.

Notice we didn't need to know the mass m of the ball to solve the problem.