Physics 153 Section T0H - Solution to Problem 11

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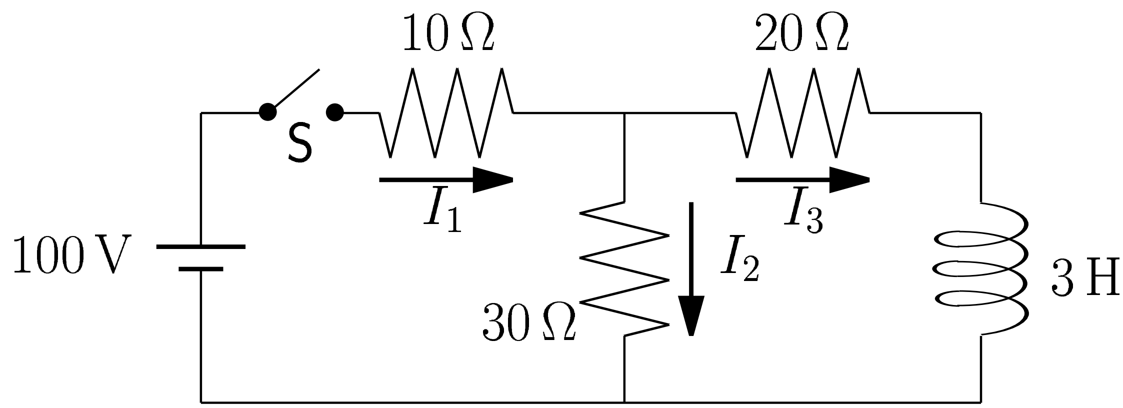
1 Assigned Problem

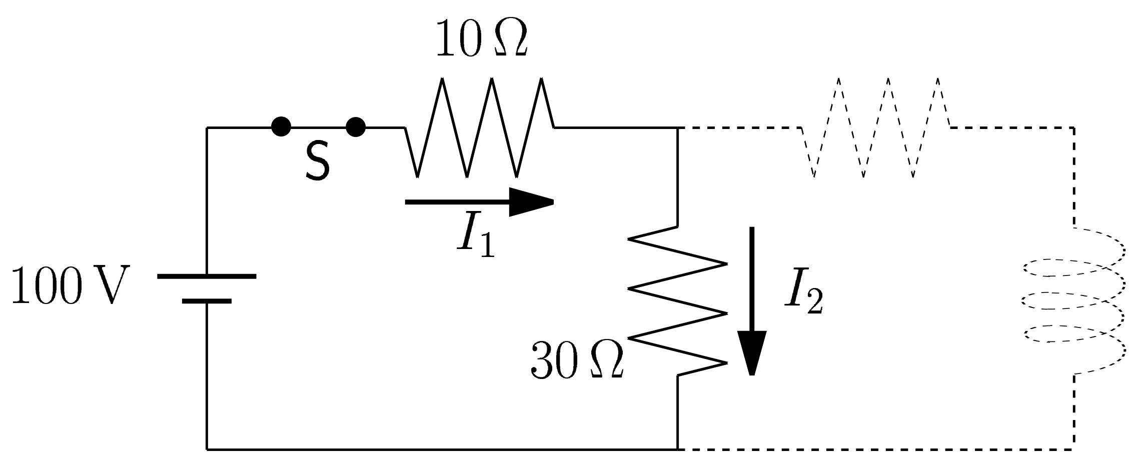


For the circuit shown above, find the currents I_1, I_2, and I_3 (a) immediately after switch S is closed and (b) a long time after switch S has been closed. After the switch has been closed for a long time, it is opened. Find the three currents (c) immediately after switch S is opened and (d) a long time after switch S was opened.

2 Solution

2.1 Part (a)

Immediately after the switch is closed $I_3(0) = 0$ because the inductor resists the change in current. So we can drop this part of the circuit and we basically just have the following circuit:

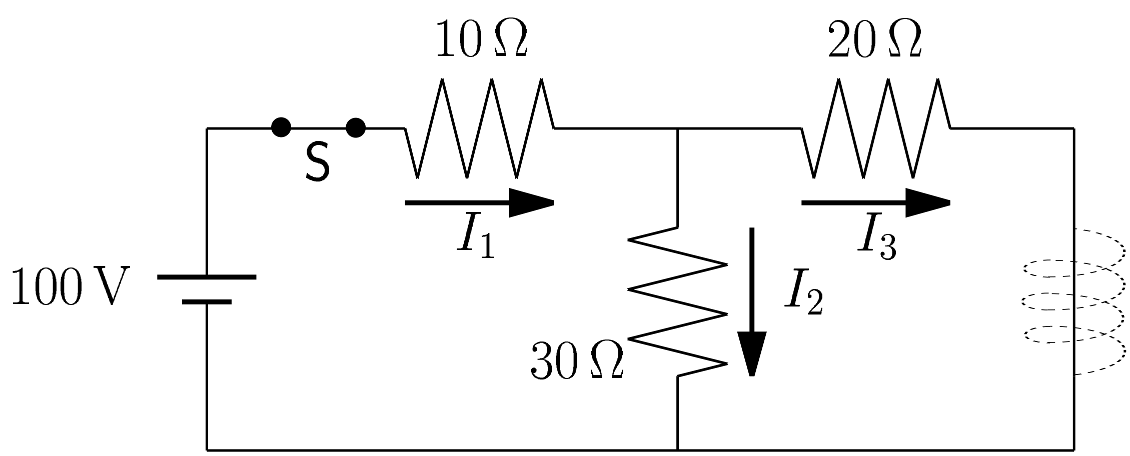


So all the current goes through R_1 and R_2 and

$$I_1(0) = I_2(0) = \frac{V}{R_1 + R_2} = \frac{100}{40} = 2.5\,\mathrm{A}. \qquad (1)$$

2.2 Part (b)

A long time after the switch has been closed the inductor just acts like a short-circuit so we basically have the following circuit:



So the resistors R_2 and R_3 are in parallel with equivalent resistance

$$R_{23} = \left[\frac{1}{R_2} + \frac{1}{R_3}\right]^{-1} = 12\,\Omega. \tag{2}$$

The current $I_1(\infty)$ is easy to calculate because we can just treat it as a battery and two resistors in series so

$$I_1(\infty) = \frac{V}{R_1 + R_{23}} = \frac{100}{22} = 4.55\,\mathrm{A}. \tag{3}$$

Using Kirchhoff's rules

$$I_1 = I_2 + I_3 \tag{4}$$

$$I_2 R_2 = I_3 R_3 \tag{5}$$

and solving for $I_2(\infty)$ and $I_3(\infty)$ give

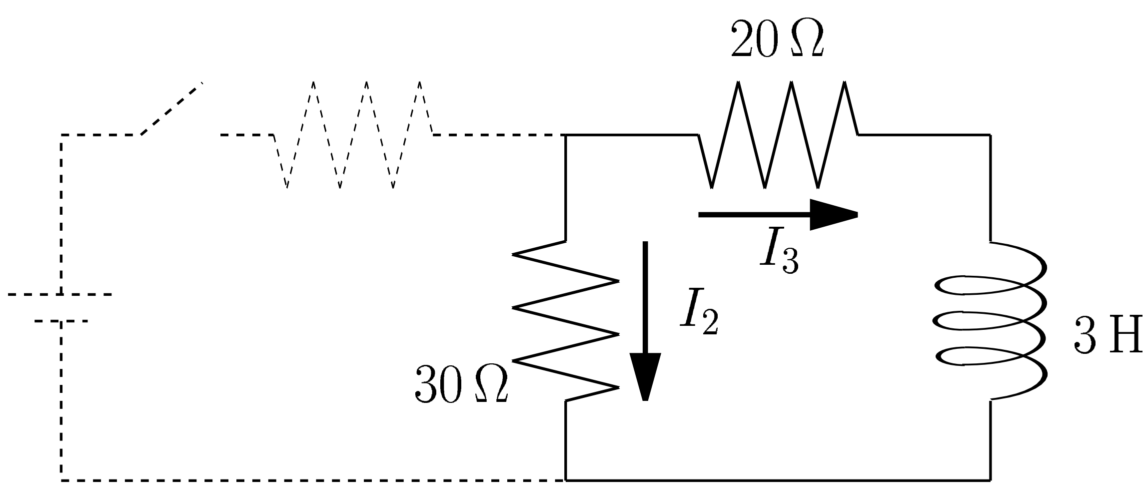
$$I_2(\infty) = \frac{R_3}{R_2 + R_3} I_1(\infty) = 1.82\,\mathrm{A} \tag{6}$$

$$I_3(\infty) = \frac{R_2}{R_2 + R_3} I_1(\infty) = 2.73\,\mathrm{A}. \tag{7}$$

2.3 Part (c)

Now we open the switch again and reset our clock to zero. I'll use primes to denote everything after the switch was opened.

Now there is no current through R_1 ($I_1'(0) = 0$) so the circuit looks like:



The inductor resists a change in the current so I_3 is the same as it was just before the switch was opened

$$I_3'(0) = I_3(\infty) = 2.73\,\mathrm{A}. \tag{8}$$

And since the circuit consists of just a single loop,

$$I_2'(0) = -I_3'(0) = -2.73\,\mathrm{A}. \tag{9}$$

The minus sign just indicates it is going in the opposite direction to what is drawn.

2.4 Part (d)

After a long time we still have the same circuit as in part (c) except the inductor becomes a short-circuit. So there is no voltage source and all the currents go to zero,

$$I_1'(\infty) = I_2'(\infty) = I_3'(\infty) = 0. \tag{10}$$

Aside: Notice we didn't need the self-inductance L anywhere. We would have needed it if we needed to calculate the time constant for any reason.