Physics 153 Section T0H - Solution to Problem 2

Rik Blok

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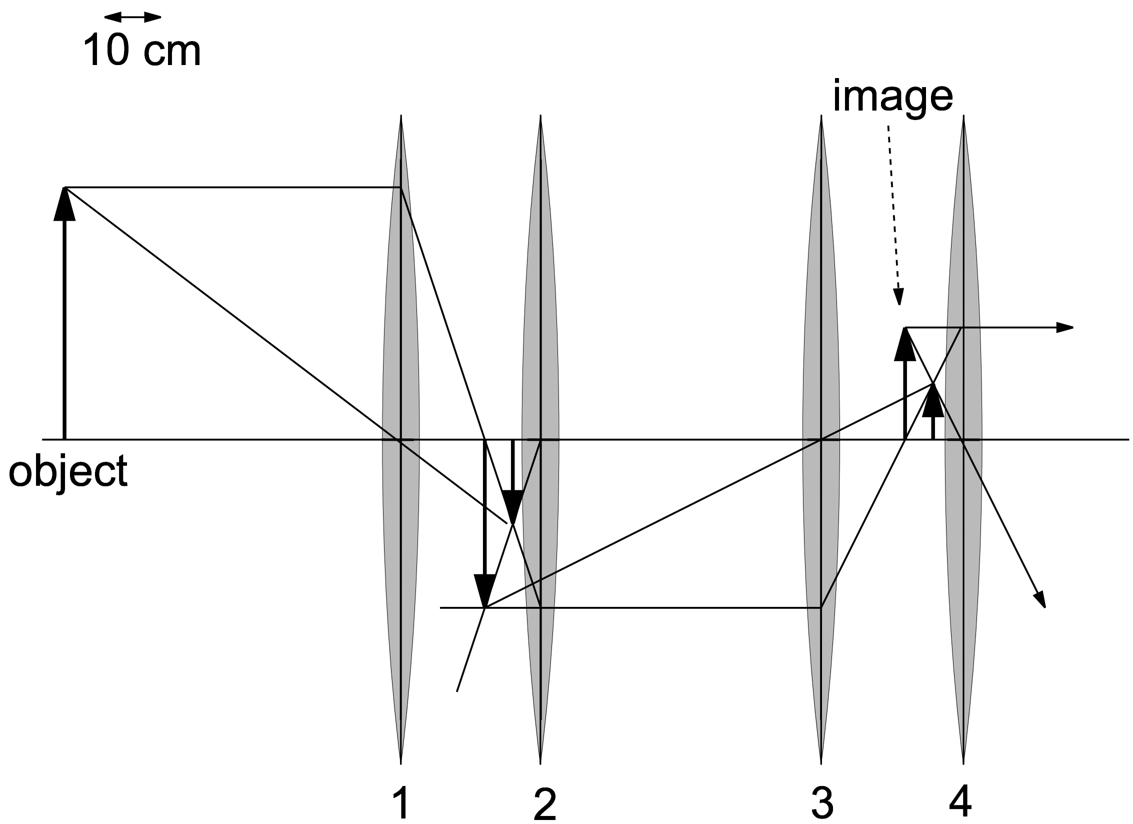
1 Assigned Problem

Two converging lenses with focal lengths $f_1 = 15\,\mathrm{cm}$ and $f_2 = 10\,\mathrm{cm}$ can be be positioned to form a simple refracting telescope by placing lens 1 $25\,\mathrm{cm}$ in front of lens 2. (*a*) Draw a ray diagram produced by an object $60\,\mathrm{cm}$ in front of lens 1 (the objective lens). The resultant image is inverted but it can be flipped by placing a second identical telescope with its objective $50\,\mathrm{cm}$ behind lens 2, producing an upright image. (*b*) Calculate the net magnification of the object. A clever TA thinks he can replace the first three of these lenses by a single lens to produce the exact same image. (*c*) Where must the lens be placed and what are the characteristics and focal length of the replacement lens? (*d*) What's wrong with this configuration? (*Hint:* try to draw the ray diagram.)

2 Solution

2.1 Part (a)

For a telescope of this type the distance between the two lenses is $f_o + f_e$ and the objective is chosen as the lens with the greater focal length (ie. $f_o = f_1 = 15\,\mathrm{cm}$). Here's the ray diagram for the full system including all four lenses:



2.2 Part (b)

First let's calculate the magnification of the first two lenses:

The first image is formed by the $f_1 = 15\,\mathrm{cm}$ lens with

$s_1 = 60\,\mathrm{cm}$ so

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} \tag{1}$$

so

$$s_1' = 20\,\mathrm{cm} \tag{2}$$

with a magnification of

$$M_1 = \frac{-s_1'}{s_1} = -1/3. \tag{3}$$

Since the separation between the two lenses is $f_1 + f_2 = 25\,\mathrm{cm}$, this makes the second object distance $s_2 = 5\,\mathrm{cm}$ and

$$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} \tag{4}$$

so

$$s_2' = -10\,\mathrm{cm} \tag{5}$$

with a magnification of

$$M_2 = \frac{-s_2'}{s_2} = 2. \tag{6}$$

Thus, the magnification of the two lenses together is

$$M_{1+2} = M_1 M_2 = -2/3. \tag{7}$$

The next telescope is positioned such that the object distance is again $s_3 = 60\,\mathrm{cm}$, and it is exactly the same configuration of lenses so the magnification will be the same:

$$M_{3+4} = M_{1+2} = -2/3 \tag{8}$$

and the net magnification of all four lenses is

$$M_{1+2+3+4} = M_{1+2} M_{3+4} = +4/9. \tag{9}$$

2.3 Part (c)

To replace the first three lenses, you want to reproduce the image of the first lens from the original object. The image and the object are separated by a distance of

$$s + s' = 155\,\mathrm{cm} \tag{10}$$

and the net magnification is

$$M = \frac{-s'}{s} = M_{1+2+3} = (-1/3)(2)(-1/3) = +2/9. \tag{11}$$

The only solution to these two equations is

$$s = 199.3\,\mathrm{cm} \tag{12}$$
$$s' = -44.3\,\mathrm{cm}. \tag{13}$$

The focal length can be calculated from the thin lens equation

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{14}$$

giving a lens with a focal length of

$$f = -57.0\,\mathrm{cm} \tag{15}$$

which is a *diverging* lens.

3 Part (d)

The problem with this configuration is that $s = 199.3\,\mathrm{cm}$ actually places the diverging lens *behind* the converging lens. Hence, the incident light will hit the converging lens first and refract in a completely different way. To convince yourself of this, draw a ray diagram.