Physics 153 Section T0H - Solution to Problem 3

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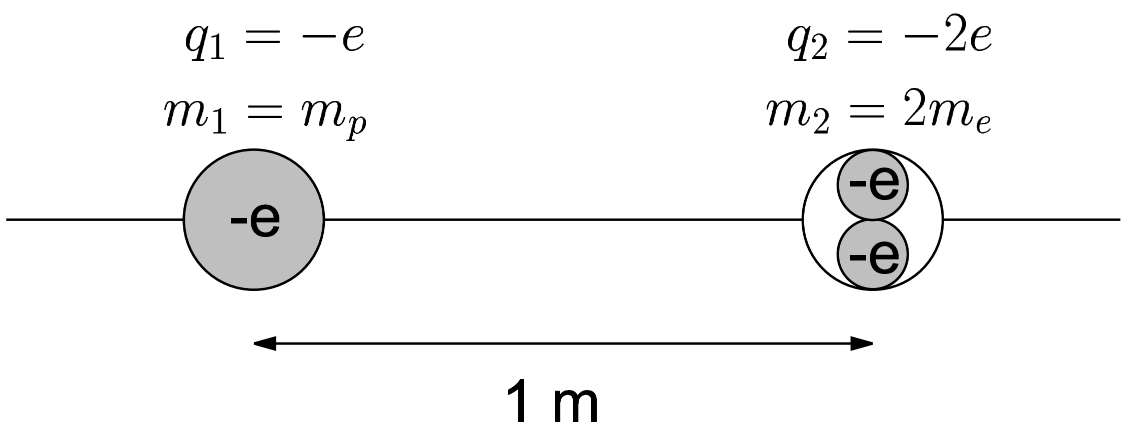
1 Assigned Problem



Two point charges, one an *anti-proton* with charge $-e$ and the other a closely-bound pair of electrons with charge $-2e$ are held fixed at a separation of $1\,\mathrm{m}$, as shown above. (*a*) At what position may a proton (charge $= +e$) be placed so that it is in electrostatic equilibrium? This configuration is actually unstable. (*b*) Describe what would actually happen to the proton and why. (*Hint:* What other force should be considered?) (*c*) Calculate the proton's initial acceleration.

2 Solution

2.1 Part (a)

To achieve equilibrium the net force on the proton must be zero. Obviously, this can only happen when it is along the same line as the two other charges. Further, the proton must be between the two charges otherwise they will both be pulling in the same direction and a balance of forces won't be possible. Therefore we only need to consider one variable x, the distance the proton is to the right of the anti-proton.

The electrostatic force on the proton q_3 due to the anti-proton q_1 is

$$F_{13} = \frac{kq_1q_3}{x^2} \tag{1}$$

$$= \frac{-ke^2}{x^2} \tag{2}$$

and the force due to the electrons q_2 is

$$F_{23} = -\frac{kq_2q_3}{(1-x)^2} \tag{3}$$
$$= \frac{2ke^2}{(1-x)^2}. \tag{4}$$

The net force on the proton (in the $+x$ direction) is $F_{13}+F_{23}$ and equilibrium exists when the net force is zero:

$$0 = F_{13} + F_{23} \tag{5}$$
$$= ke^2\left[\frac{-1}{x^2} + \frac{2}{(1-x)^2}\right] \tag{6}$$

which has the solutions

$$x_1 = +0.41\,\mathrm{m} \tag{7}$$
$$x_2 = -2.41\,\mathrm{m}. \tag{8}$$

Only the solution $x_1 = +0.41\,\mathrm{m}$ makes sense because then the proton is held in place by the equal pulls of the two negative charges.

Aside: The other solution is just an artifact of the calculation. If we had proceeded more carefully we would have written (in full vector notation) $\mathbf{F}_{13} = (kq_1q_3/x^2)\hat{\mathbf{r}}_{13}$ where $\hat{\mathbf{r}}_{13}$ would be $+1$ for $x > 0$ and -1 for $x < 0$. Then we would have to solve two separate problems: one for $x > 0$ and another for $x < 0$...but we'd have gotten the same result, namely that there is only one equilibrium at $x = +0.41\,\mathrm{m}$.

2.2 Part (b)

In part (a) only electrostatic forces were considered but although weak, the gravitational force should not be neglected. The contribution from each particle is

$$\begin{aligned} F_{13,g} &= -\frac{Gm_1m_3}{x^2} && (9) \\ &= -1.1066 \times 10^{-63}\,\mathrm{N} && (10) \end{aligned}$$

and

$$F_{23,g} = \frac{Gm_2m_3}{x^2} \quad (11)$$
$$= 5.8 \times 10^{-67}\,\mathrm{N} \quad (12)$$

so there is a *very* small net gravitational attraction towards the anti-proton.

As a result the proton will accelerate towards the anti-proton and eventually collide with it, annihilating both particles with a flash.

2.3 Part (c)

Initially, the electrostatic forces are completely balanced so the only net force is the difference between the gravitational forces

$$F_{3,net} = -1.106 \times 10^{-63}\,\mathrm{N} \quad (13)$$

and the net acceleration is

$$a_3 = F_{3,net}/m_3 = -6.63 \times 10^{-37}\,\mathrm{m/s^2}. \qquad (14)$$

Realistically, this acceleration is much too small to be significant. The presence of any other matter in the vicinity would bias the experiment (through other, unanticipated, forces) and the results may be completely different. But, in the ideal, the gravitational force would pull the proton out of electrostatic equilibrium and, once out of equilibrium, the electrostatic force itself would begin pulling the proton towards the anti-proton.