Physics 153 Section T0H - Week 4

Continuous Charge Distributions

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1 Review

Recall, the force on a test charge q in a field $\mathbf{E}$ is

$$\mathbf{F}_q = q\mathbf{E}. \tag{1}$$

If the source of the field is simply a point charge Q at a distance r then

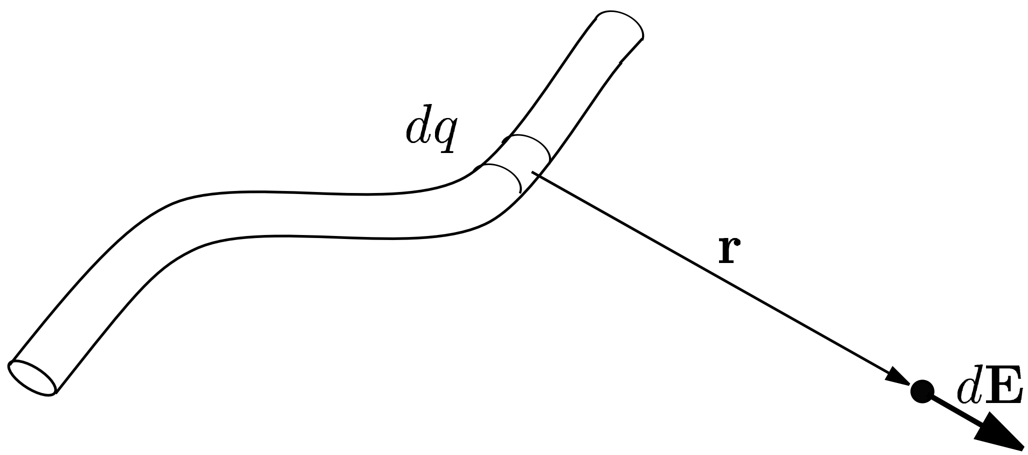
$$\mathbf{E} = \frac{kQ}{r^2}\hat{\mathbf{r}}. \tag{2}$$

2 Continuous Charge Distributions

There are two ways to calculate E-fields with continuous charge distributions: integration and Gauss' Law.

This week we will discuss integration.

The idea is to split the continuum into many small point charges and add up the contribution from each.

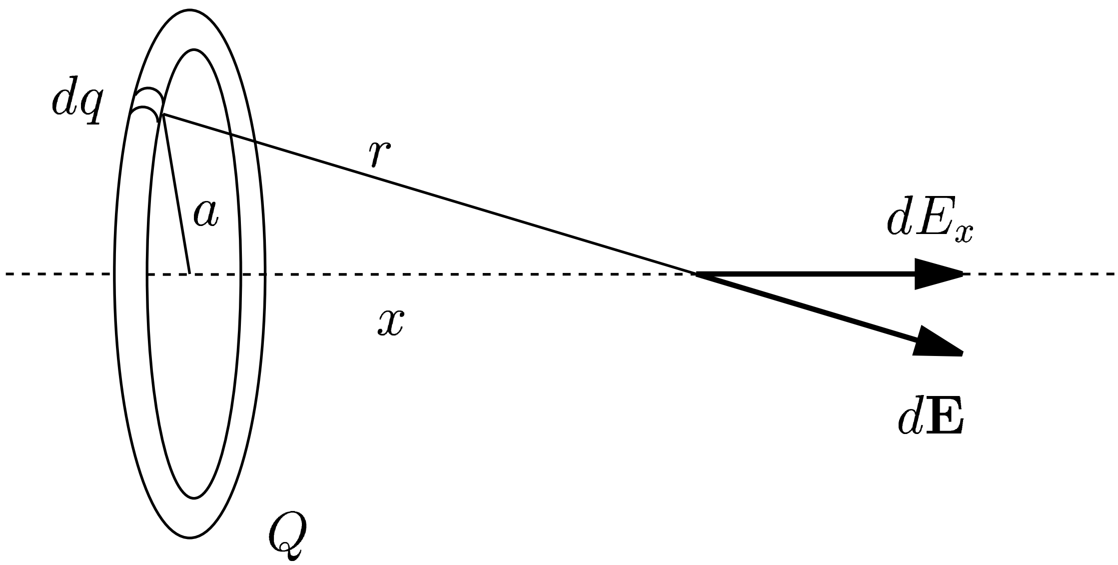


The total E-field is

$$\mathbf{E} = \int d\mathbf{E} \tag{3}$$

$$= \int \frac{k\ dq}{r^2}\hat{\mathbf{r}}. \tag{4}$$

3 Example: $\mathbf{E}$ on the axis of a ring charge



$$\mathbf{E} = \int d\mathbf{E} \tag{5}$$

Only components of $\mathbf{E}$ in x-direction add up because perpendicular components cancel out, so

$$\mathbf{E} = \int dE_x \, \hat{\mathbf{i}}. \tag{6}$$

From similar triangles

$$\frac{dE_x}{|d\mathbf{E}|} = \frac{x}{r} \tag{7}$$

so

$$dE_x = |d\mathbf{E}| \, \frac{x}{r}. \tag{8}$$

From Coulomb's law

$$|d\mathbf{E}| = \frac{k \, dq}{r^2} \tag{9}$$

so this all comes together to give

$$\begin{aligned} \mathbf{E} &= \int \frac{k \, dq}{r^2} \frac{x}{r} \, \hat{\mathbf{i}} && (10) \\ &= \frac{kx}{r^3} \int dq \, \hat{\mathbf{i}}. && (11) \end{aligned}$$

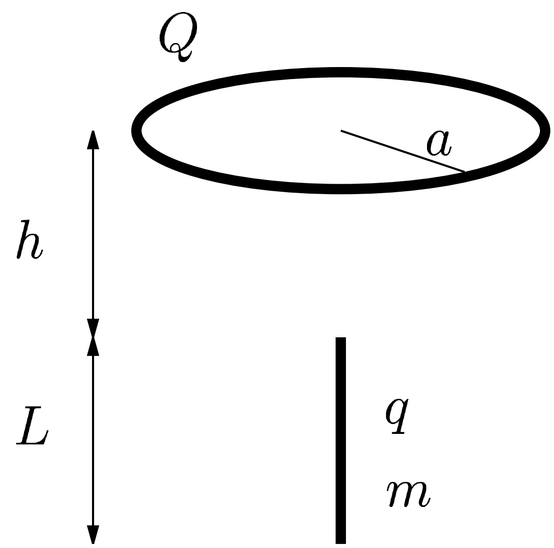
Now the integral is just over all dq so it must sum up to the total charge Q (exercise for the reader) and

$$\mathbf{E} = \frac{kQx}{r^3}\,\hat{\mathbf{i}}. \tag{12}$$

Finally, we use $r = \sqrt{x^2 + a^2}$ to give the familiar form

$$\mathbf{E} = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}\,\hat{\mathbf{i}}. \tag{13}$$

4 Assigned Problem



In a laboratory, a ring of charge $Q = 8\,\mu\mathrm{C}$ with radius $a = 30\,\mathrm{cm}$ levitates a charged rod of length $L = 20\,\mathrm{cm}$ and mass $m = 100\,\mathrm{g}$, as shown above. The top of the floating rod is $h = 10\,\mathrm{cm}$ below the center of the ring. (a) Draw a free body diagram for the rod in equilibrium. (b) Assuming it is uniformly distributed, what is the total charge q of the rod?