Physics 153 Section T0H - Week 5

Gauss's Law

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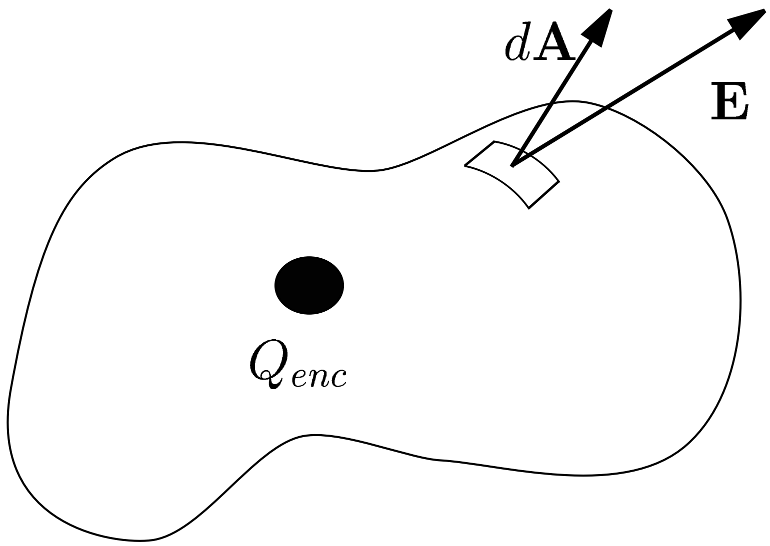
1 Gauss's Law

Gauss's law allows you to calculate the electric field at the surface of some imaginary object by summing up all the charge contained within the object

$$\int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} = Q_{enc}/\epsilon_0 \tag{1}$$

where $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2$.



In the equation $\mathbf{E}$ is the electric field strength at a point on the surface and $d\mathbf{A}$ is a tiny area of that surface with a direction which is pointing *straight out* of the surface.

Application of Gauss's law can be tricky and only works when the surface is chosen very carefully.

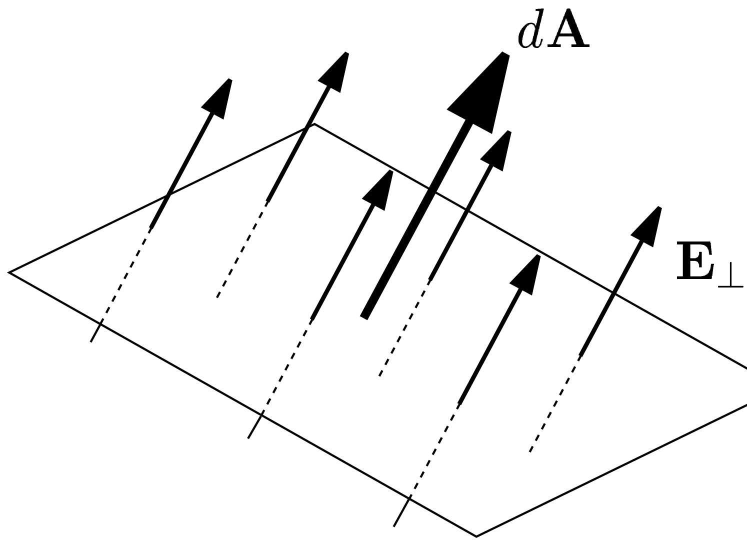
2 Gaussian surfaces

The trick is that you want to choose a surface that simplifies the calculation. The easiest way I've found is to construct surfaces which consist of sections (faces) which are always either *parallel* or *perpendicular* to the electric field.

Then you can split Gauss's law into

$$
\begin{aligned}
Q_{enc}/\epsilon_0 &= \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} \qquad (2) \\
&= \int_{\text{parallel}} \mathbf{E}_{\parallel} \cdot d\mathbf{A} + \int_{\text{perpendicular}} \mathbf{E}_{\perp} \cdot d\mathbf{A}. \qquad (3)
\end{aligned}
$$

2.1 Perpendicular field

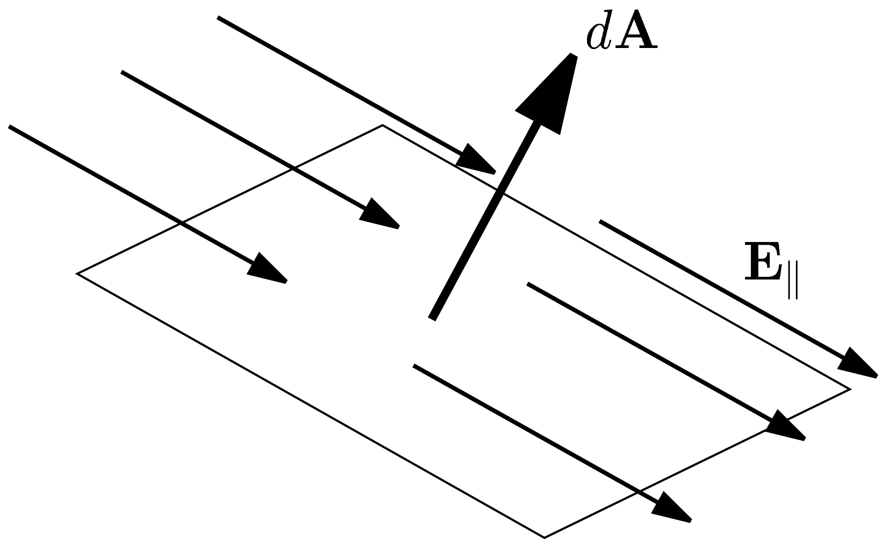


If $\mathbf{E}_\perp$ is perpendicular to the surface (note: I mean the surface itself, not the unit vector $d\mathbf{A}$) then $E_\perp$ is constant everywhere along that surface and

$$\int_{\text{perpendicular}} \mathbf{E}_\perp \cdot d\mathbf{A} = E_\perp A \tag{4}$$

so you don't have to solve an integral at all!

2.2 Parallel field



If $\mathbf{E}_{\parallel}$ is parallel to the surface (again, the surface itself, not the unit vector $d\mathbf{A}$) then the dot product of $\mathbf{E}_{\parallel} \cdot d\mathbf{A}$ is zero everywhere so

$$\int_{\text{parallel}} \mathbf{E}_{\parallel} \cdot d\mathbf{A} = 0. \tag{5}$$

Again, no integral to solve!

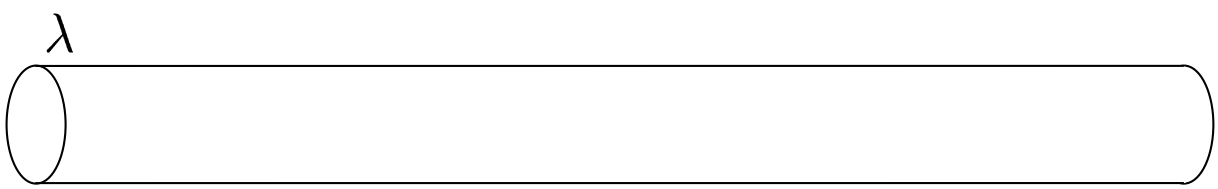
3 Symmetry

The tricky part about the above discussion is that you have to know something about the electric field just to use Gauss's law to solve for the field!

You can make statements about which way the electric field is pointing in a problem by imagining how you could rotate, shift, or flip the system such that it would look exactly the same. Then the electric field lines must look exactly the same under the same transformations.

4 Example: A line charge

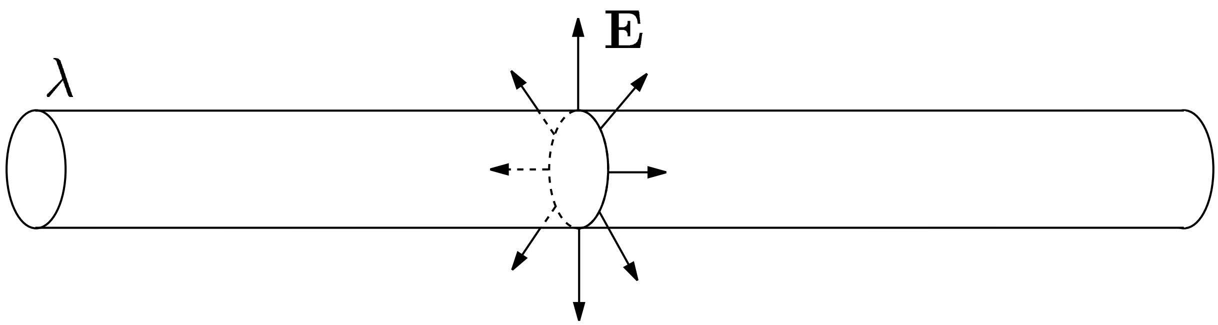
4.1 Problem



Calculate the electric field around a very long line of charge with linear charge density λ.

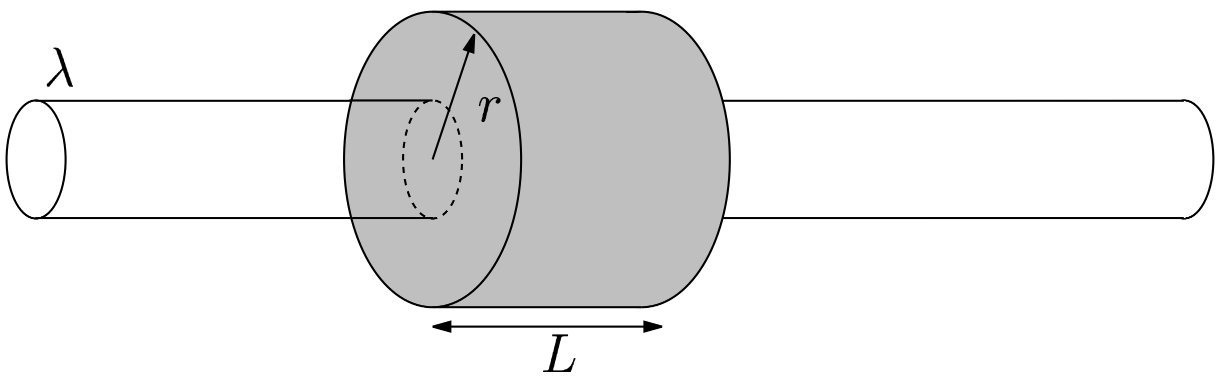
4.2 Solution

First we look for the symmetry of the problem. The bit about a “very long line” means we should assume the line is infinite so shifting along the axis shouldn't make any difference. Nor should rotating around the axis or flipping it. Hopefully you can convince yourself (it gets easier with practice...) that the only electric field lines that wouldn't be affected by any of these transformations are lines which point radially away from the axis. (it gets easier with practice...)



Now that we know the electric field points radially away from the line, we want to construct a surface such that all the faces are either parallel or perpendicular to the electric field.

Obviously, the surface must be a coaxial cylinder (with some radius r and length L). Then the side of the cylinder is perpendicular to the electric field and the ends are parallel.



Now it gets easy...the total charge enclosed within the surface is

$$Q_{enc} = \lambda L \tag{6}$$

so Gauss's law gives

$$\begin{aligned} Q_{enc}/\epsilon_0 &= \int \mathbf{E} \cdot d\mathbf{A} && (7) \\ \lambda L/\epsilon_0 &= E_\perp A_{\text{side}} + (E_\| A_{\text{ends}}) \cdot 0 && (8) \\ &= E_\perp 2\pi r L + 0. && (9) \end{aligned}$$

Solving for the electric field gives

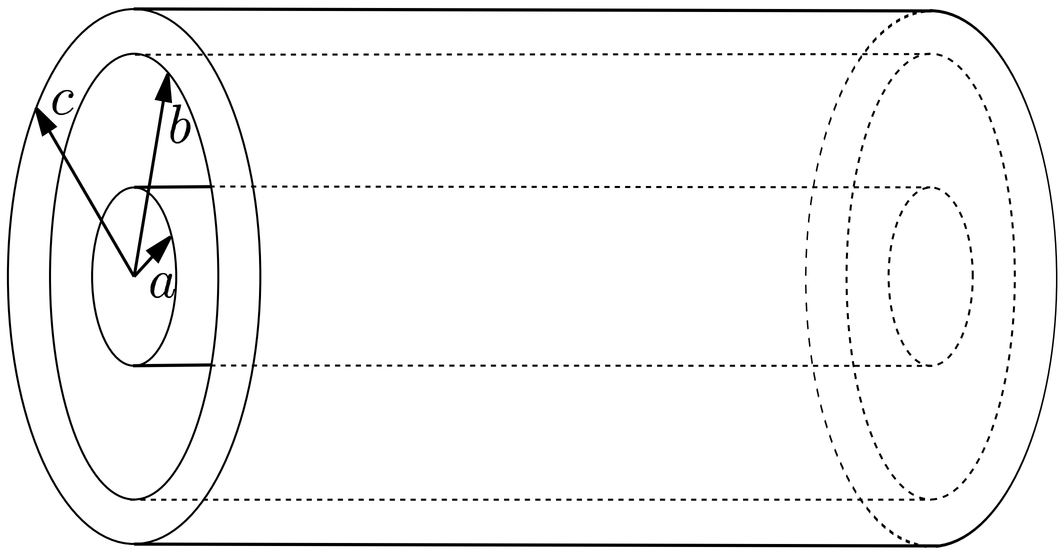
$$E(r) = E_{\perp} = \frac{\lambda}{2\pi\epsilon_0 r}. \tag{10}$$

5 Conductors

There is only one simple rule you have to remember when working with electric fields and conductors: the (static) electric field is zero everywhere inside a conductor

$$\mathbf{E} = \mathbf{0}. \tag{11}$$

6 Assigned Problem



A very long, uniformly charged cylinder with volume charge density ρ and radius a is coaxial with a conducting cylindrical shell with inner radius b and outer radius c, as shown above. (a) Calculate and plot the electric field strength E as a function of radius r. (b) What is the surface charge density σ on the inner surface of the shell ($r = b$)?