Physics 153 Section T0H - Week 6

Capacitors

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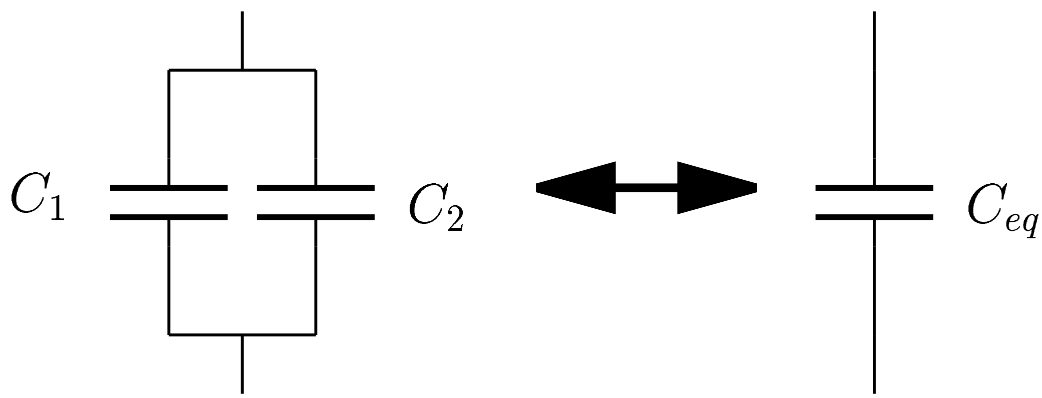
# 1 Comments

Notes on web page (`http://www.physics.ubc.ca/~blok/phys153/`) are now in PDF and Postscript format. If you have problems with them, let me know.

I will keep this week's tutorial simple to try and encourage more students to do the problem.

# 2 Capacitors in circuits

## 2.1 Parallel

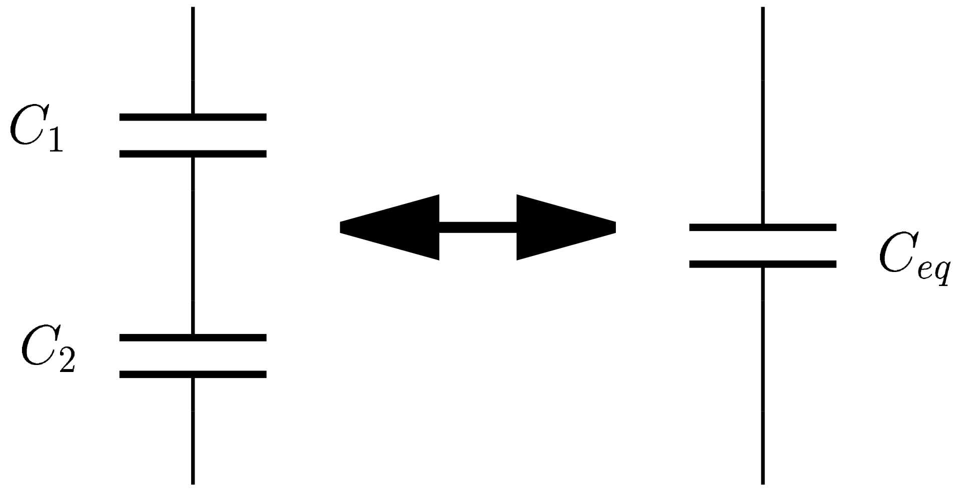



When two capacitors are connected in parallel, they are equivalent to of a single capacitor with capacitance

$$C_{eq,\text{parallel}} = C_1 + C_2. \tag{1}$$

(Extends to any number of capacitors.)

## 2.2 Series



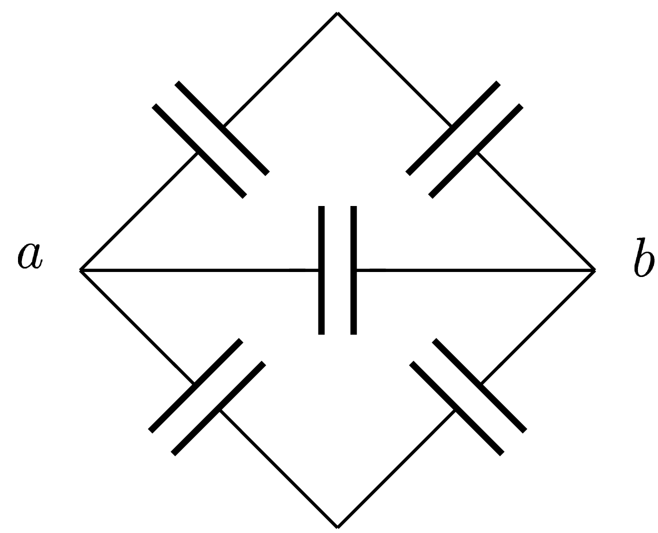

When two capacitors are connected in series, they are equivalent to of a single capacitor with capacitance

$$\frac{1}{C_{eq,\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{2}$$

(Extends to any number of capacitors.)

# 3 Example

(From Tipler Ch. 21 #49.)

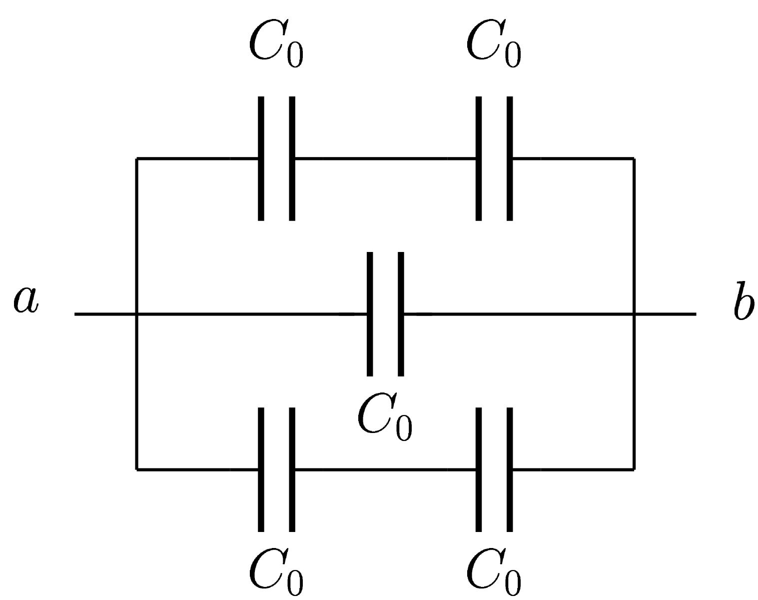



Five identical capacitors of capacitance $C_0$ are connected in a bridge network as shown above. (a) What is the equivalent capacitance between points $a$ and $b$? (b) Find the equivalent capacitance if the capacitance between $a$ and $b$ is changed to $10C_0$.

# 4 Solution

## 4.1 Part (a)

The first thing we can do is redraw the diagram so it looks more familiar without changing anything...

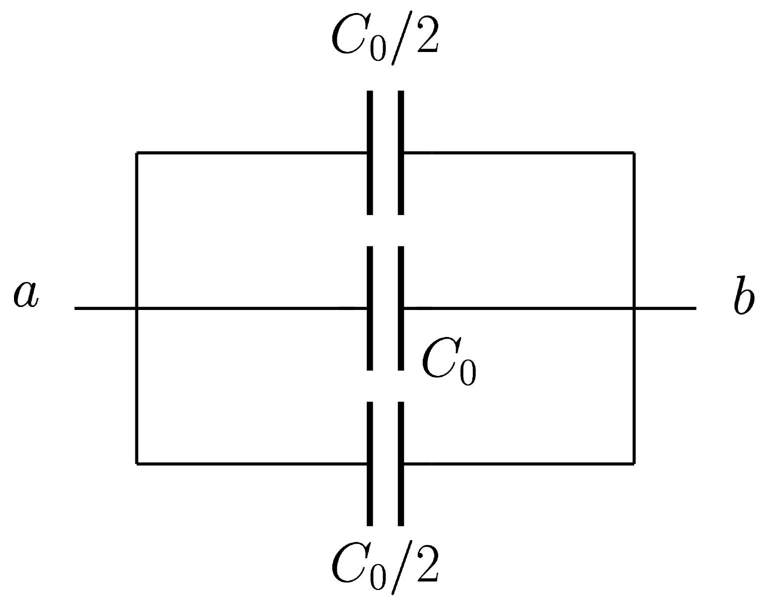


Now, we can't find any set of capacitors which are all parallel to each other but the top and bottom of the diagram each have two capacitors in series. The equivalent capacitance for each of these combinations is

$$C_{\text{top}} = C_{\text{bottom}} = \left[\frac{1}{C_0} + \frac{1}{C_0}\right]^{-1} \tag{3}$$

$$= [2/C_0]^{-1} \tag{4}$$

$$= C_0/2. \tag{5}$$




So the circuit is equivalent to the above diagram. Now we notice that we are just dealing with three capacitors, all in parallel so the equivalent capacitance is just

$$
\begin{aligned}
C_{eq} &= C_0 + C_{\text{top}} + C_{\text{bottom}} && (6)\\
&= C_0 + C_0/2 + C_0/2 && (7)\\
&= 2C_0. && (8)
\end{aligned}
$$

## 4.2 Part (b)

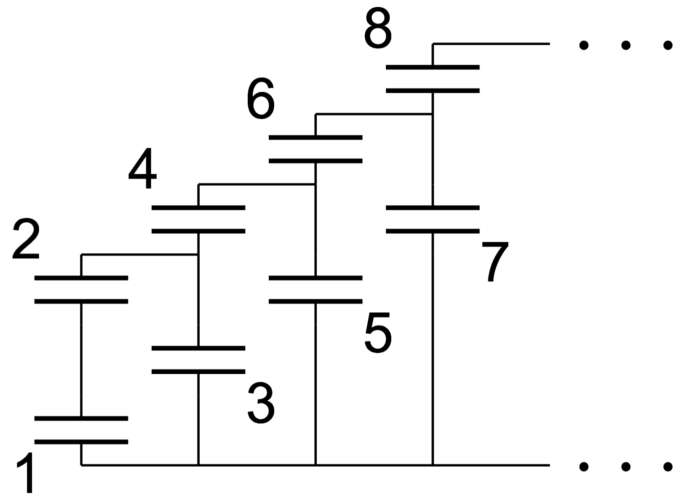
The bit about "the capacitance between $a$ and $b$ is changed to $10C_0$" is not very clear but they just mean the middle capacitor is changed.

So the calculation of $C_{\text{top}}$ and $C_{\text{bottom}}$ still stand giving the exact same diagram as the last one we saw except the middle capacitor is now $10C_0$.

So, again we have three capacitors in parallel, giving

$$\begin{aligned} C_{eq} &= 10C_0 + C_{\text{top}} + C_{\text{bottom}} && (9) \\ &= 10C_0 + C_0/2 + C_0/2 && (10) \\ &= 11C_0. && (11) \end{aligned}$$

# 5 Assigned Problem




A group of identical $C = 1\,\mu\mathrm{F}$ capacitors are connected recursively, as shown above. Calculate the equivalent capacitance of (a) the first two capacitors, (b) the first three capacitors. (c) Repeat, up to the eighth capacitor. (d) (*Optional.*) In the limit of infinitely many capacitors, what is the equivalent capacitance of this configuration? (*Hint*: $\lim_{n\to\infty} C_{eq}(n) = C_{eq}(n+2)$.)