Physics 153 Section T0H - Week 7
Resistivity

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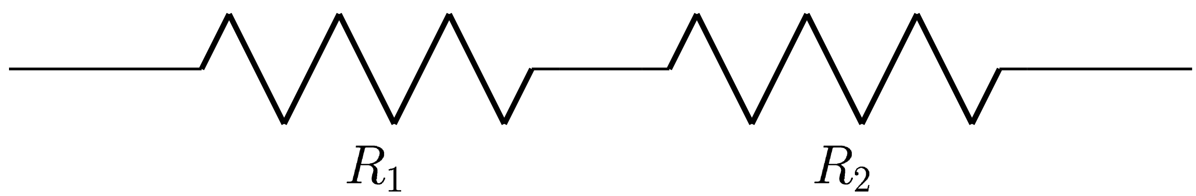
1 Resistors in circuits

1.1 Series

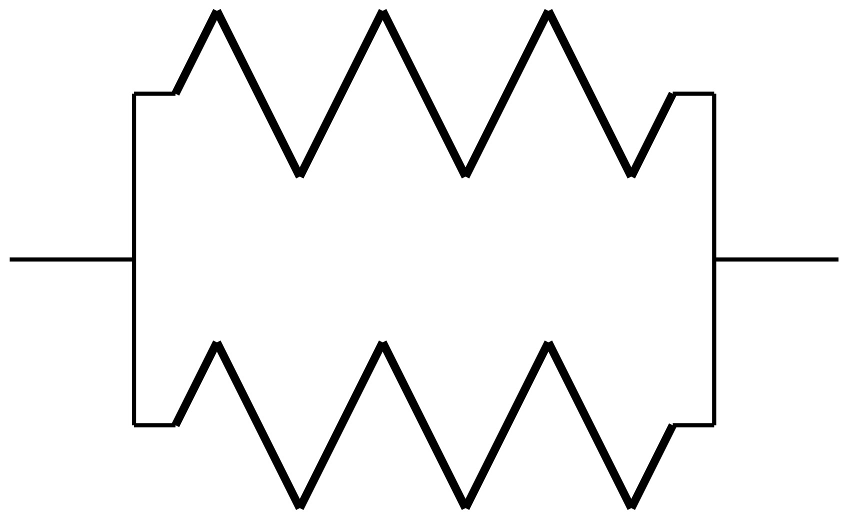


When two resistors are connected in series, they are equivalent to of a single resistor with resistance

$$R_{eq,\text{series}} = R_1 + R_2. \tag{1}$$

(Extends to any number of resistors.)

1.2 Parallel



When two resistors are connected in parallel, they are equivalent to of a single resistor with capacitance

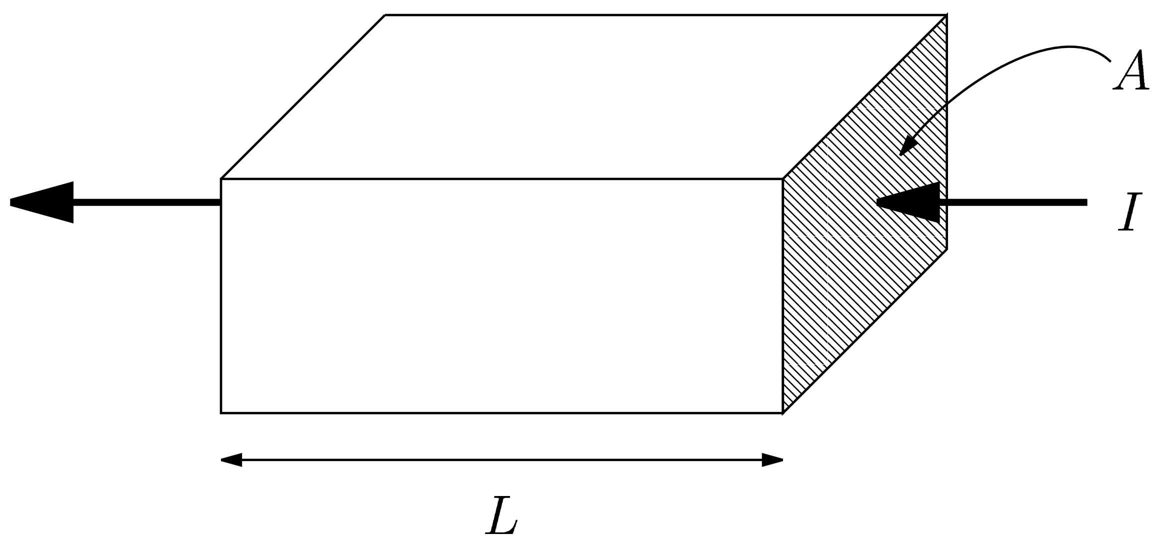
$$\frac{1}{R_{eq,\mathrm{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2}$$

(Extends to any number of resistors.)

2 Resistivity

Electrical resistance between two ends of a solid object depends on the distance L the electrons have to travel, the cross-sectional area A of the surface, and the resistivity ρ (*rho*) of the material

$$R = \rho \frac{L}{A}. \tag{3}$$



3 Calculating resistance

To calculate the total resistance of a (non-rectangular) object from the resistivity you need to divide the object into individual chunks over which both the length L and area A are fixed.

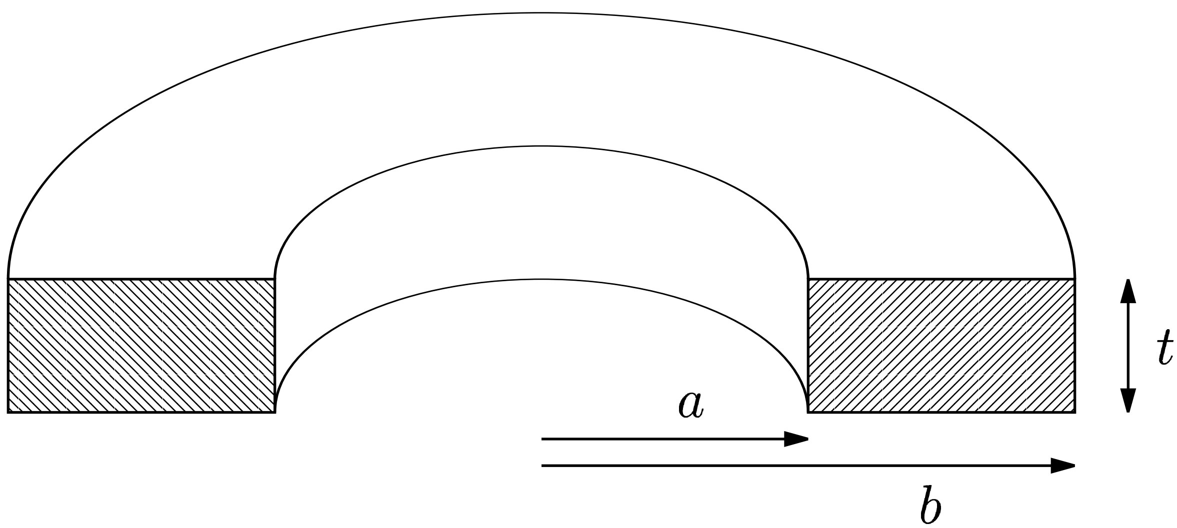
Then each "chunk" is a resistor so now just calculate the equivalent resistance (in series or parallel) of the entire object.

In many cases you will need to divide the object into infinitesimal pieces and solve an integral.

Remember, L is in the direction of the current and A is cross-sectional.

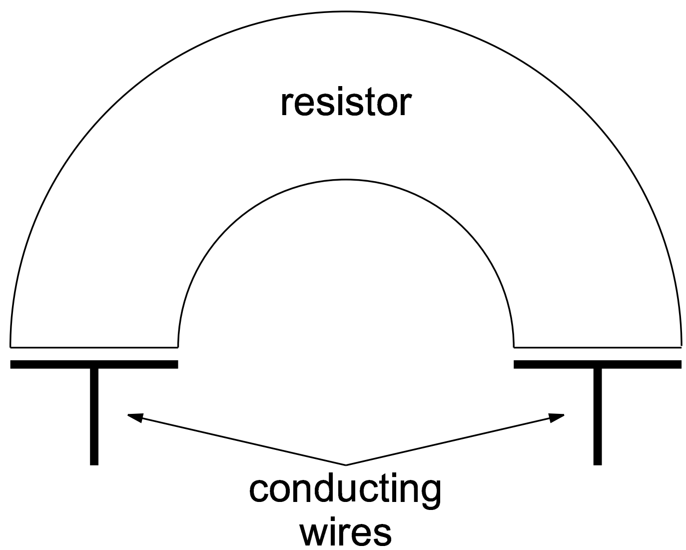
4 Example

(From Tipler Ch. 22 #65.)



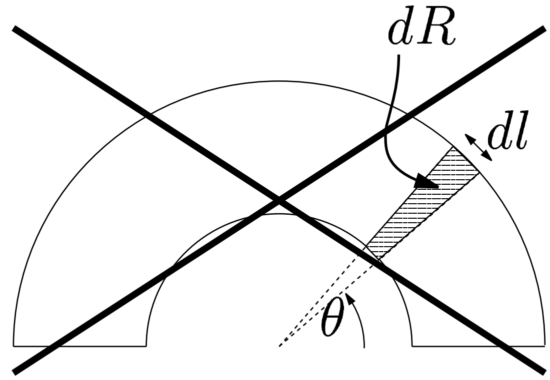
Find the resistance between the ends of the half ring shown above. The resistivity of the ring is ρ.

5 Solution



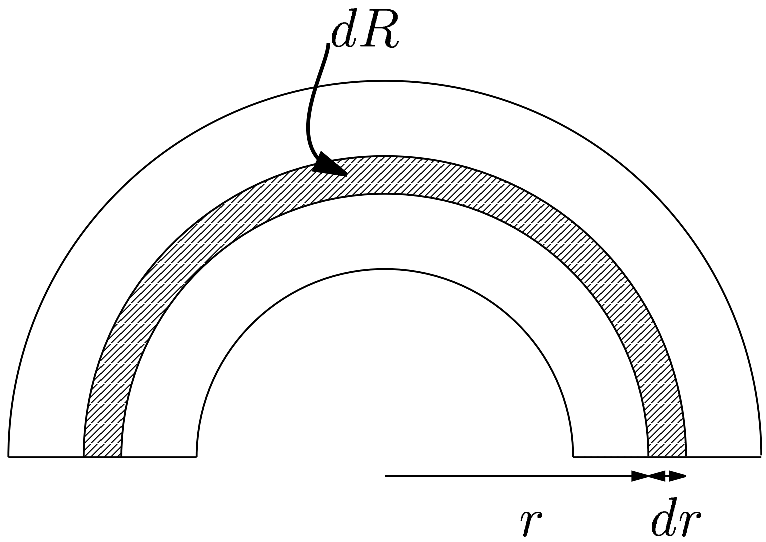
Schematically, we are trying to calculate the resistance of the resistor shown above.

It may seem natural to divide the object into pie-slice chunks as shown below. Then you would just have a bunch of resistors (each chunk) in series and you could add them up to get $R = \int dR$ the total resistance.



But it won't work! It fails because the length $L = dl$ isn't a constant so you can't use $dR = \rho\, dl/A$. It is smaller at the inner radius than the outer.

So we need to try something else. Let's divide the object as shown below.



Now we are dealing with a bunch of pieces (resistors) which are side-by-side, or in parallel. So the equivalent resistance is given by

$$\frac{1}{R} = \int \frac{1}{dR} \tag{4}$$

where each little resistor has resistance

$$dR = \rho \frac{L}{dA}. \tag{5}$$

The cross-sectional area of each resistor is just $dA = t\,dr$ and the length is $L = \pi r$ so

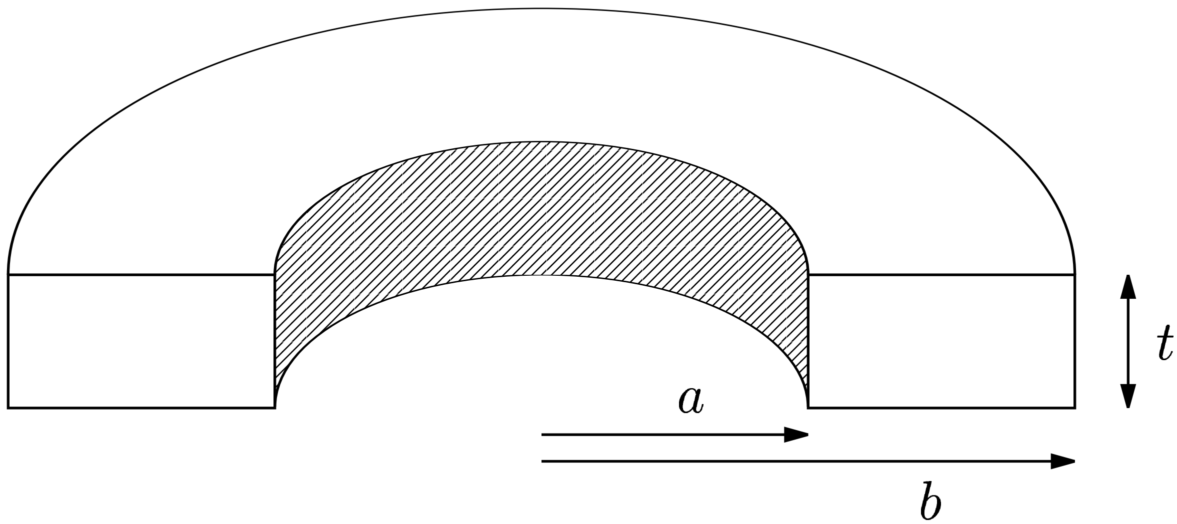
$$\begin{aligned} \frac{1}{R} &= \int \frac{dA}{\rho L} && (6)\\ &= \int_a^b \frac{t\,dr}{\rho \pi r} && (7)\\ &= \frac{t}{\rho\pi}\int_a^b \frac{dr}{r} && (8)\\ &= \frac{t}{\rho\pi}\ln(b/a). && (9) \end{aligned}$$

So, the final answer is

$$R = \frac{\rho\pi}{t\ln(b/a)}. \quad (10)$$

(*Note:* My notation was kind of flawed because each element dR was actually *infinite*, not infinitesimal. The things that are infinitesimal are the contributions $1/dR$. Ignore this bit if you don't know what I'm talking about.)

6 Assigned Problem



Find the resistance between the inner (shaded) and outer surfaces of the half ring shown above. The resistivity of the ring is ρ.