Physics 153 Section T0H - Week 9

Ampère's Law

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1 Oops!

Last week I said you could solve for the time constant in Tipler Ch. 23 #60 (assigned tutorial problem) by just neglecting the $600\,\Omega$ resistor because initially all the current just went through the other one.

But it turns out you can't. Actually, some of you had the right intuition: you have to treat the resistors as if they are in parallel and calculate the equivalent resistance.

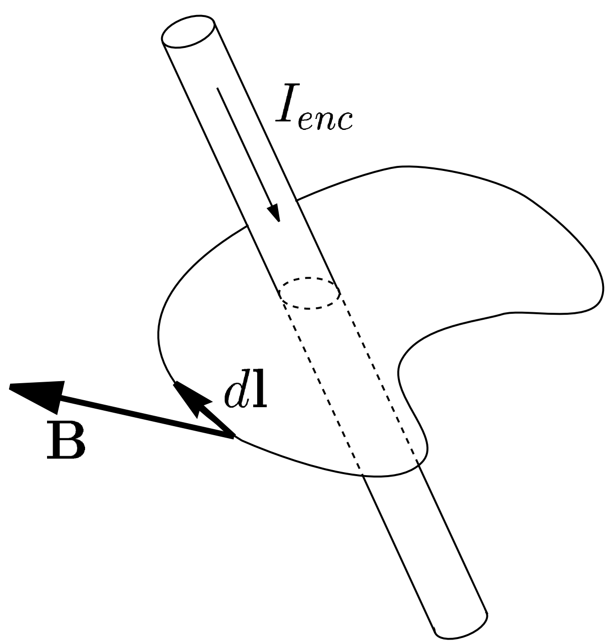
To see why, you have to actually solve the differential equation. You won't be expected to know how to do this but I put it in the solution on the web page (`www.physics.ubc.ca/~blok/phys153`) in case you're curious (bored).

2 Ampère's law

Ampère's law let's you calculate the magnetic field along some imaginary loop by summing up all the currents passing through the loop

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc} \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$.



I_{enc} is the total current passing through the loop. The loop is broken into tiny sections of length dl and the contributions of the magnetic field parallel to each section is summed over all the sections. The funny $\oint$ symbol just means 'integrate over the whole loop".

3 Symmetry

Like in Gauss's law, you want to choose loops that simplify the problem. The way to do that is to guess the direction of the magnetic field from the *Right Hand Rule* and then break the Ampèrian loop into sections which are always parallel or always perpendicular to the magnetic field.

Then you can split Ampère's law into

$$\mu_0 I_{enc} = \oint \mathbf{B} \cdot d\mathbf{l} \tag{2}$$

$$= \int_{\text{parallel}} \mathbf{B}_{\parallel} \cdot d\mathbf{l} + \int_{\text{perpendicular}} \mathbf{B}_{\perp} \cdot d\mathbf{l}. \tag{3}$$

If the vectors $\mathbf{B}$ and $d\mathbf{l}$ are perpendicular then the dot product is zero so we can drop that term out of the equation.

If they are parallel then the dot product is just $\mathbf{B} \cdot d\mathbf{l} = B\,dl$ and B will have a constant magnitude so

$$\mu_0 I_{enc} = \int_{\text{parallel}} \mathbf{B}_{\parallel} \cdot d\mathbf{l} \tag{4}$$

$$= \sum_{\text{parallel}} B \int dl \tag{5}$$

$$= \sum_{\text{parallel}} Bl. \tag{6}$$

So we don't have to solve any integrals! We just add up Bl over all parallel sections of the loop.

4 Example

(From Tipler Ch. 25 #33.)

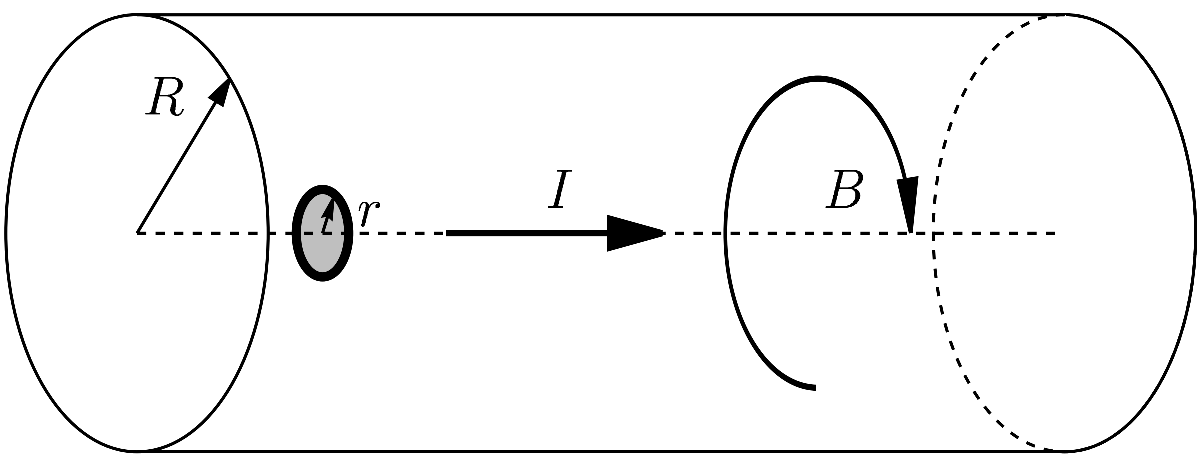
A wire of radius $0.5\,\mathrm{cm}$ carries a current of $100\,\mathrm{A}$ that is uniformly distributed over its cross-sectional area. Find B (a) $0.1\,\mathrm{cm}$ from the center of the wire, (b) at the surface of the wire, (c) at a point outside the wire $0.2\,\mathrm{cm}$ from the surface of the wire. (d) Sketch a graph of B versus the distance from the center of the wire.

5 Solution

5.1 Part (a)

"Find B $0.1\,\mathrm{cm}$ from the center of the wire."

Let's imagine the wire running to the right as shown below. Then, from the right hand rule we know the magnetic field will be clockwise circles around the wire.



So the easiest Ampèrian loop we can use is one which is always parallel to the magnetic field...a circle at a radius $r = 0.1\,\mathrm{cm}$.

The current going through the loop (the shaded area) is just a fraction of the total current in the wire. The current density in the wire (current per unit area) is

$$J = \frac{I}{\pi R^2} \tag{7}$$

(where $R = 0.5\,\mathrm{cm}$) so the total current passing through the loop is

$$I_{enc} = J(\pi r^2) = I\frac{r^2}{R^2}. \tag{8}$$

The magnetic field is constant in strength and always parallel to the loop so $\mathbf{B} \cdot d\mathbf{l} = B\,dl$ and Ampère's law reduces to

$$\begin{aligned} \mu_0 I_{enc} &= \oint \mathbf{B} \cdot d\mathbf{l} && (9) \\ &= B \oint dl && (10) \\ &= B(2\pi r). && (11) \end{aligned}$$

Substituting in for I_{enc} gives the magnetic field

$$B(r < R) = \frac{\mu_0 I_{enc}}{2\pi r} \tag{12}$$

$$= \frac{\mu_0 I}{2\pi R^2} r \tag{13}$$

and plugging in the numbers

$$B(0.1) = \frac{\mu_0 (100)}{2\pi (0.5)^2}(0.1) = 0.8\,\mathrm{mT}. \tag{14}$$

5.2 Part (b)

"Find B at the surface of the wire."

The same argument and equations apply except now $r = R$ so

$$B(r = R) = \frac{\mu_0 I}{2\pi R} \tag{15}$$

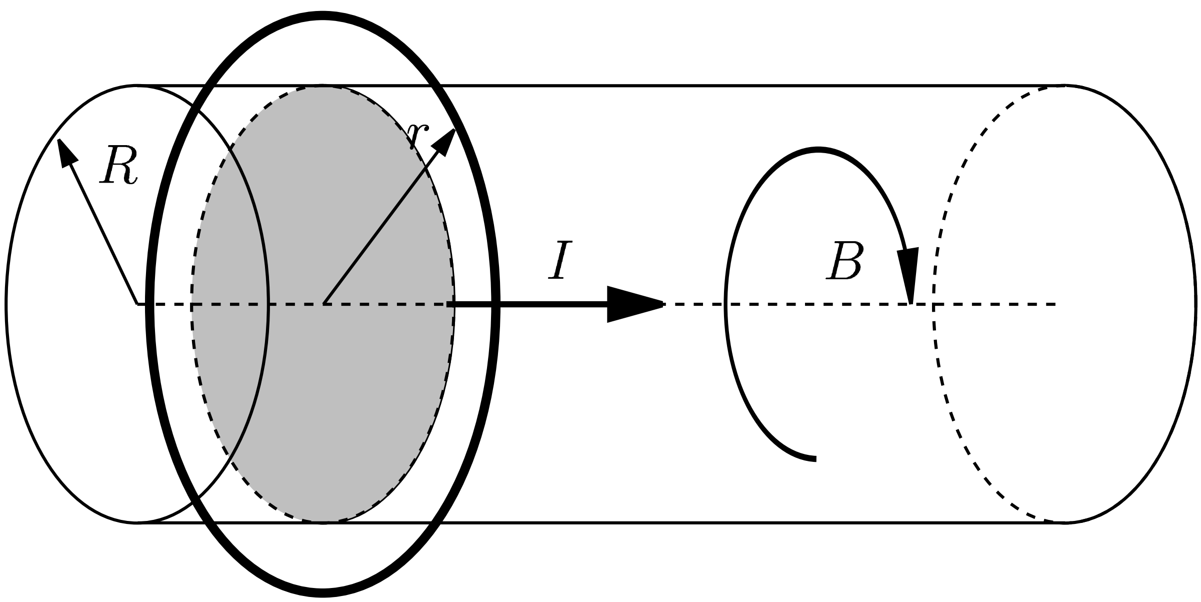
and plugging in the numbers again, gives

$$B(0.5) = \frac{\mu_0 (100)}{2\pi (0.5)} = 4\,\mathrm{mT}. \tag{16}$$

5.3 Part (c)

"Find B at a point outside the wire $0.2\,\mathrm{cm}$ from the surface of the wire."

Now we construct an Ampèrian loop with radius $r = R + 0.2\,\mathrm{cm} = 0.7\,\mathrm{cm}$.



Applying Ampère's law again gives

$$\mu_0 I_{enc} = \oint \mathbf{B} \cdot d\mathbf{l} \tag{17}$$

$$= B \oint dl \tag{18}$$

$$= B(2\pi r). \tag{19}$$

The total enclosed current is just $I_{enc} = I$ so this becomes

$$B(r > R) = \frac{\mu_0 I}{2\pi r} \tag{20}$$

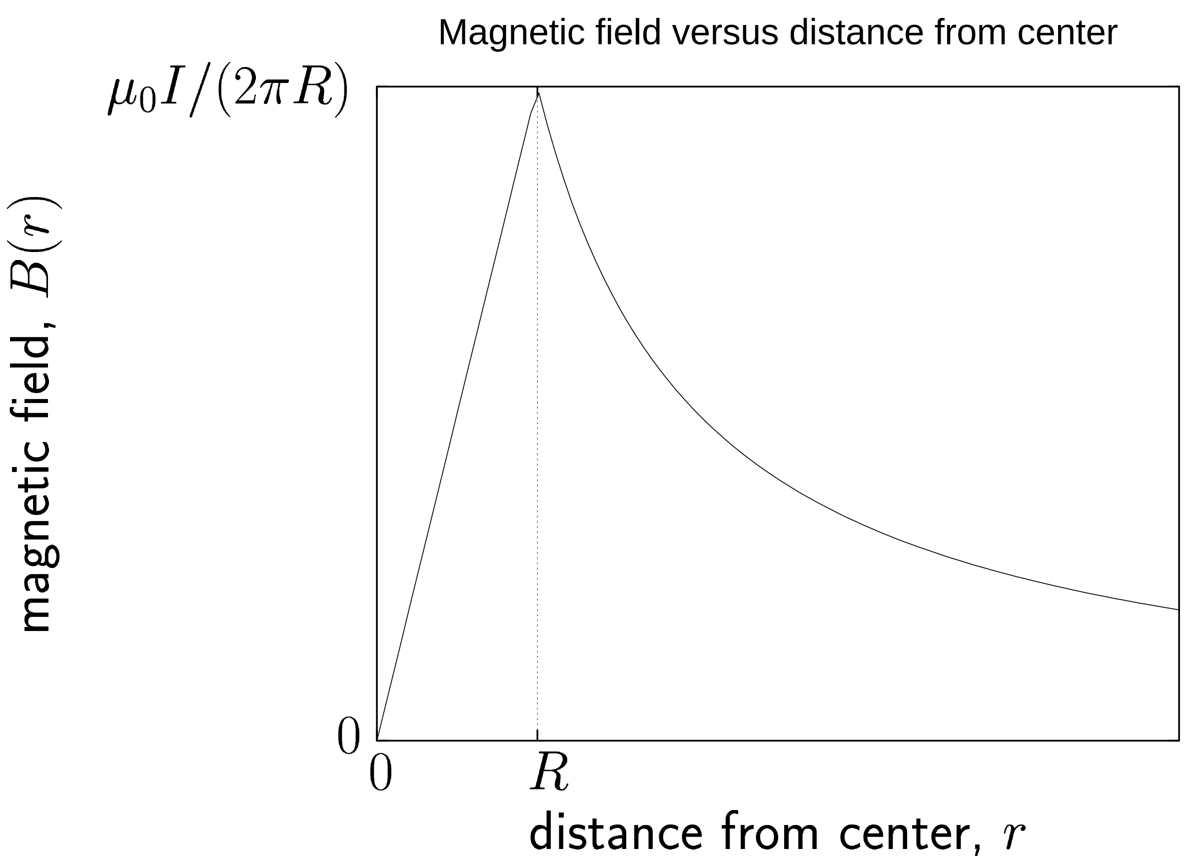
and plugging in the numbers

$$B(0.7) = \frac{\mu_0(100)}{2\pi(0.7)} = 2.86\,\mathrm{mT}. \tag{21}$$

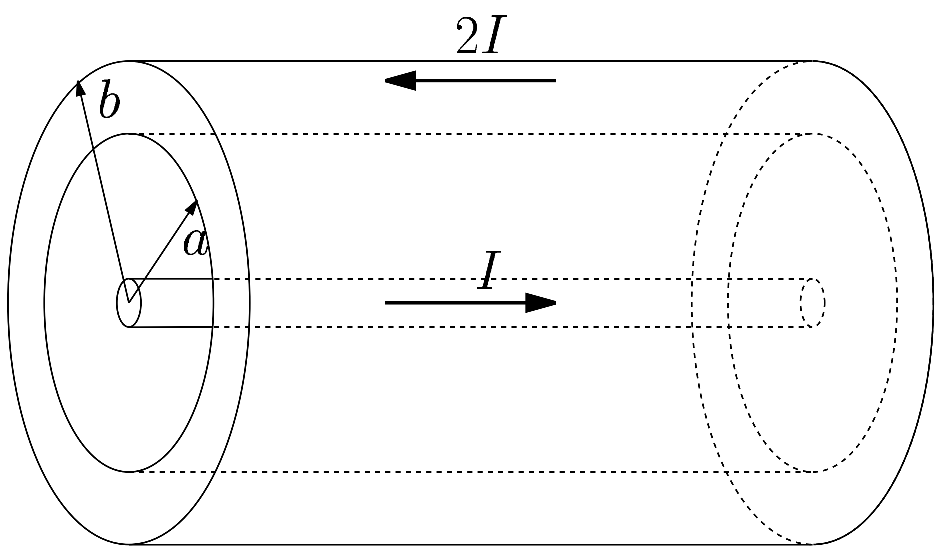
5.4 Part (d)

"Sketch a graph of B versus the distance from the center of the wire."

We've already solved for $B(r)$ in Eqs. 13, 15, and 20 so we just need to draw them, like this:



6 Assigned Problem



A very long coaxial cable consists of a thin inner wire and a concentric outer cylindrical shell with inner radius a and outer radius b. A current I runs down the inner wire and a current $2I$ comes back up the shell (uniformly distributed). Find the magnetic field $\mathbf{B}$ at a distance r from the center for (a) $r < a$, (b) $a < r < b$, and (c) $r > b$. (d) Sketch a graph of B versus r.