Final Ph.D. Oral
On the nature of the stock market: Simulations and experiments

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1 Why study the stock market?

Market is a strongly-interacting, many-particle, non-equilibrium system. 'Nuff said.

Markets have anomalous properties. Example: Were long believed to have normal fluctuations. Mandelbrot [1] discovered price fluctuations exhibited scaling: returns

$$r_\Delta(t) \equiv \log \frac{p(t)}{p(t-\Delta)} \tag{1}$$

have power law distribution tail

$$\Pr(r) \sim \frac{1}{r^{\alpha+1}} \tag{2}$$

over orders of magnitude of sampling interval Δ.

Scaling exponent is universal, $\alpha \approx 1.4$ [2].

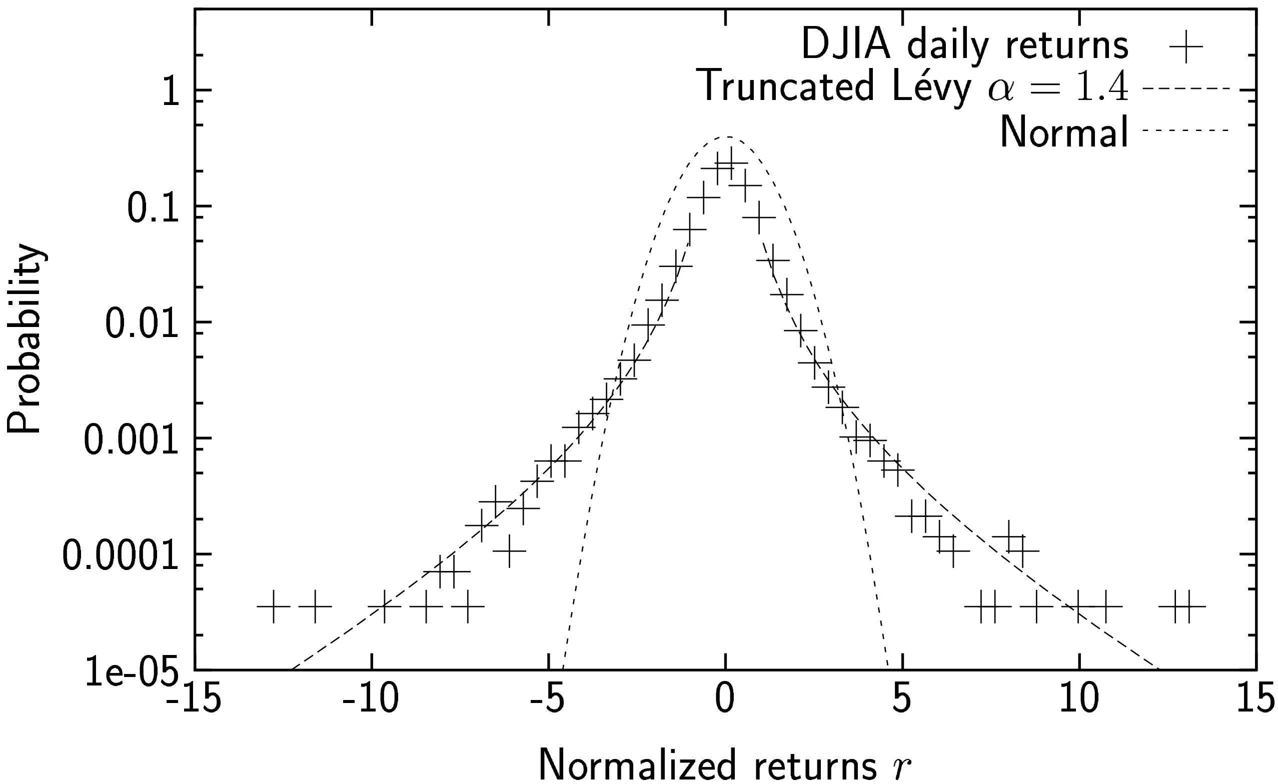
Power law and universality are suggestive of a critical phase transition.

2 What is empirically known?

2.1 Fat tails

Distribution of returns falls off much more slowly than a Gaussian so frequency of large fluctuations much more common than might be expected.

For example, daily returns of the Dow Jones Industrial Average for the last hundred years [3]:

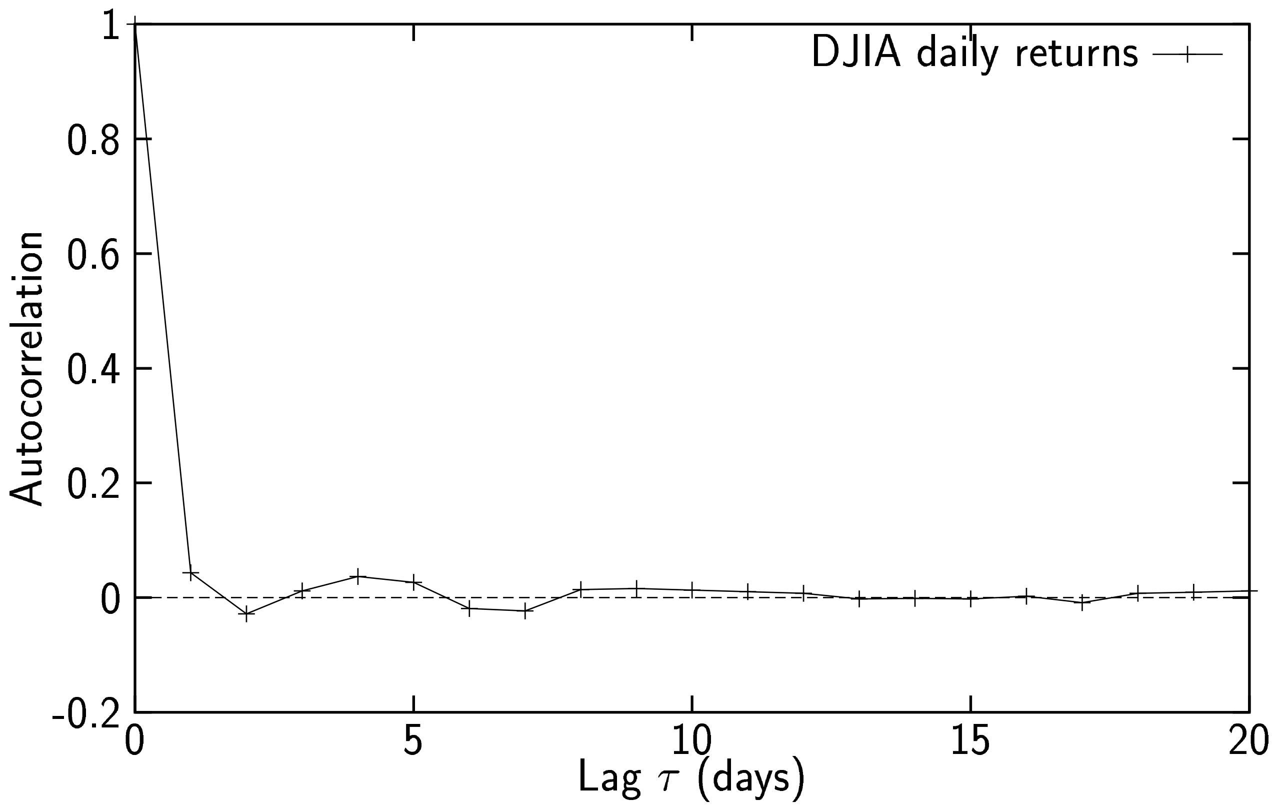


Best described by a truncated Lévy flight with power law tails which are attenuated.

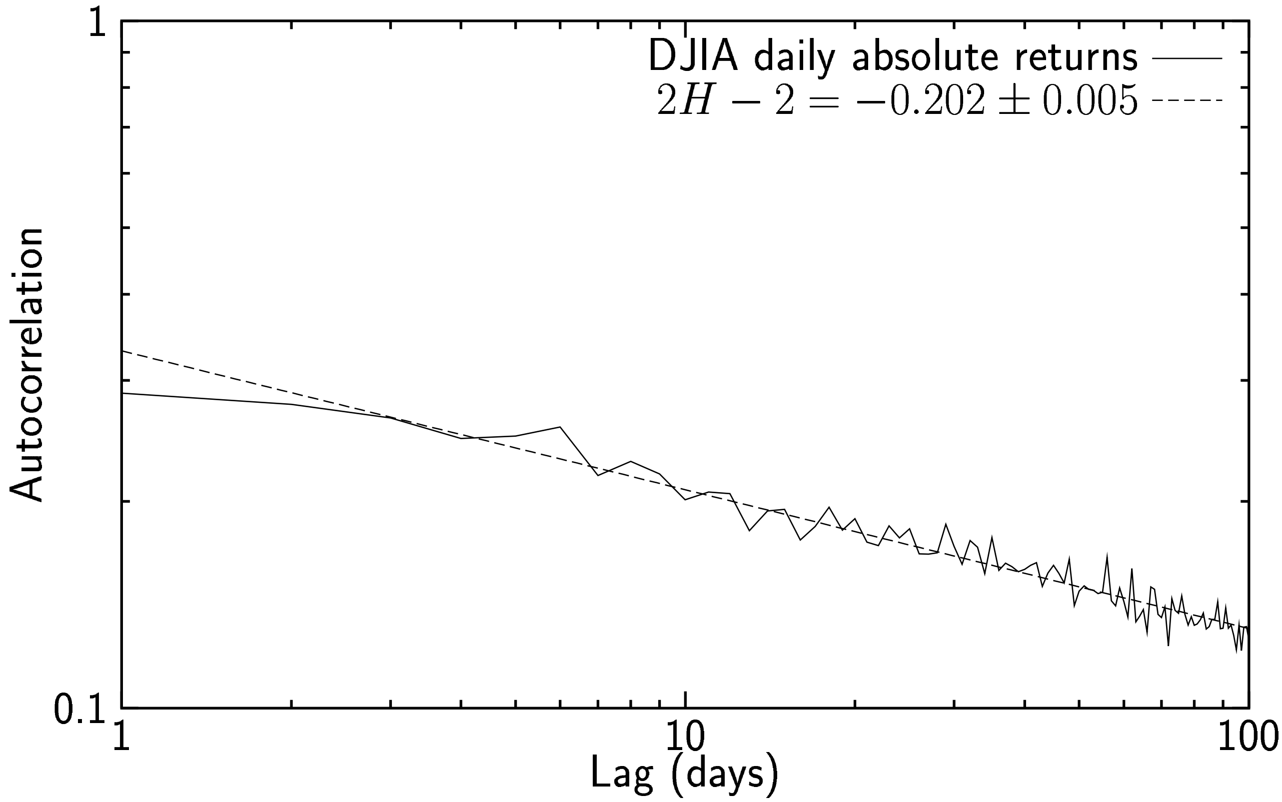
Frequency of a 10-standard deviation magnitude event is about once every 100 years, but for Gaussian is once every 10^{20} years.

2.2 Memory

No autocorrelation of returns. Can't predict movements from history:



But absolute values of returns are correlated; autocorrelation falls off as power law (much slower than exponential):



Hurst exponent $H \approx 0.9$ (very strong correlations).

Means large fluctuations tend to be clumped together—*clustered volatility*. Given a large movement can expect another one, but can't predict direction.

So the market does have a memory but can't use it to get rich!

3 How is the market modeled?

Built two models, CSEM and DSEM. Will focus on DSEM here.

N individual "agents" trading on a market with one stock.

Agents trade shares for cash and price emerges from their decisions.

Goal is to keep a certain fraction i of wealth invested in stock [4].

Ideal investment fraction affected by stochastic news releases and price fluctuations. Define evidence [5]

$$e \equiv \log \left[\frac{i}{1-i} \right]. \tag{3}$$

Then on Gaussian news release $\eta \sim N(0,1)$ evidence changes by

$$\Delta e \propto \eta. \tag{4}$$

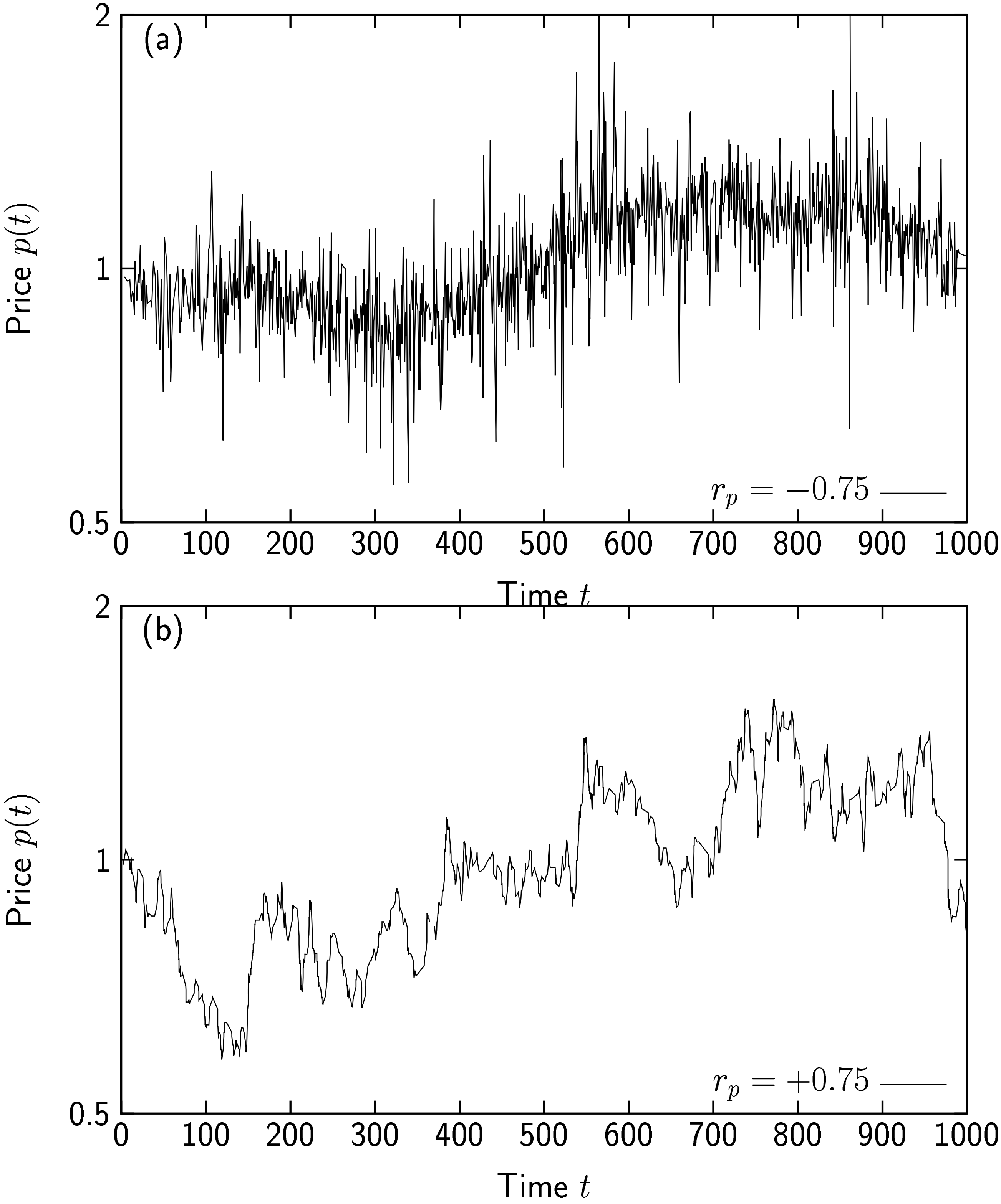
(Note driving force doesn't have fat tails or memory.)

Also respond to price fluctuations: on price change $p \to p'$

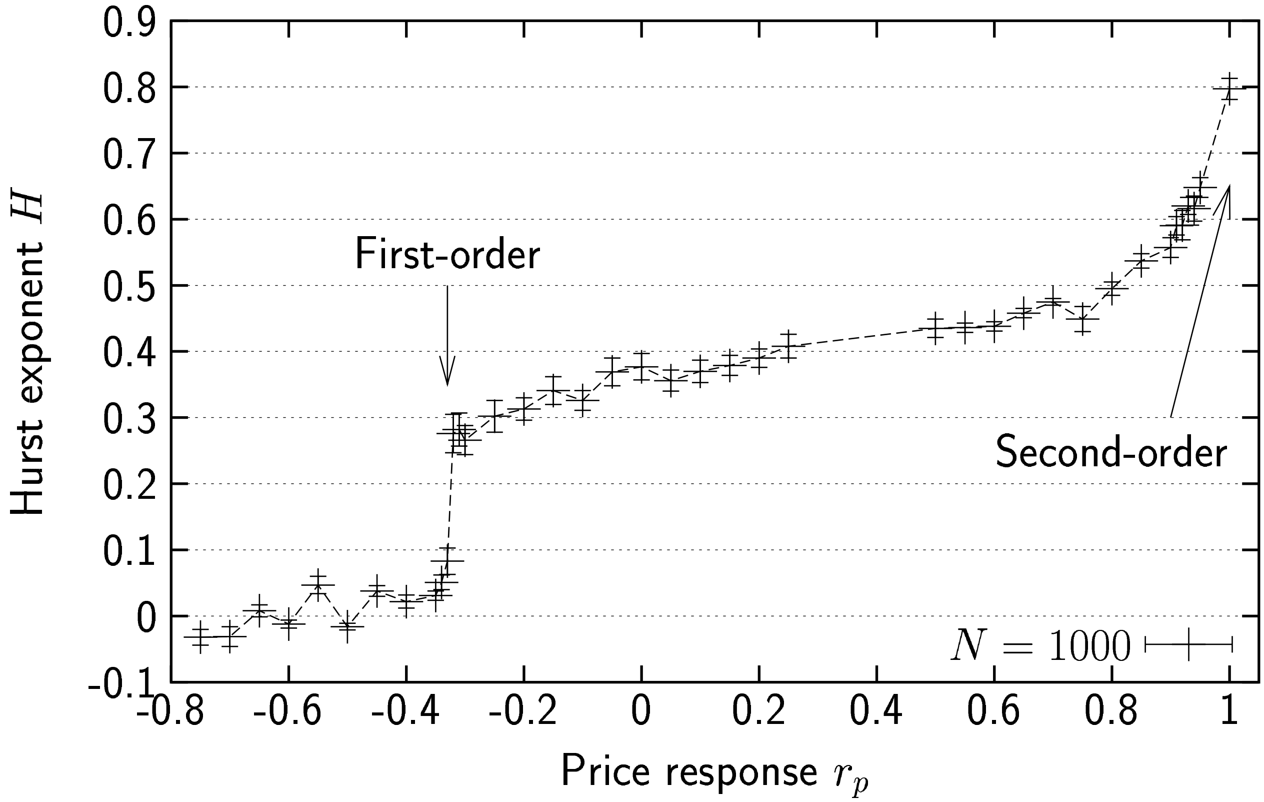
$$\Delta e = r_p \log \frac{p'}{p}. \tag{5}$$

Strength of response to price movements characterized by parameter r_p. Like confidence in price movement. (Can be positive or negative.)

4 Where's the physics?



In DSEM interesting parameter is price response r_p which sets memory.



Two phase transitions:

First-order: Discrete jump at $r_p \approx -1/3$. (Local, nucleation, intermittency.)

Second-order: Goes to $H = 1$ for $r_p \geq 1$. Critical point. (Global, correlations span entire system, cascades.)

Fitting to power law

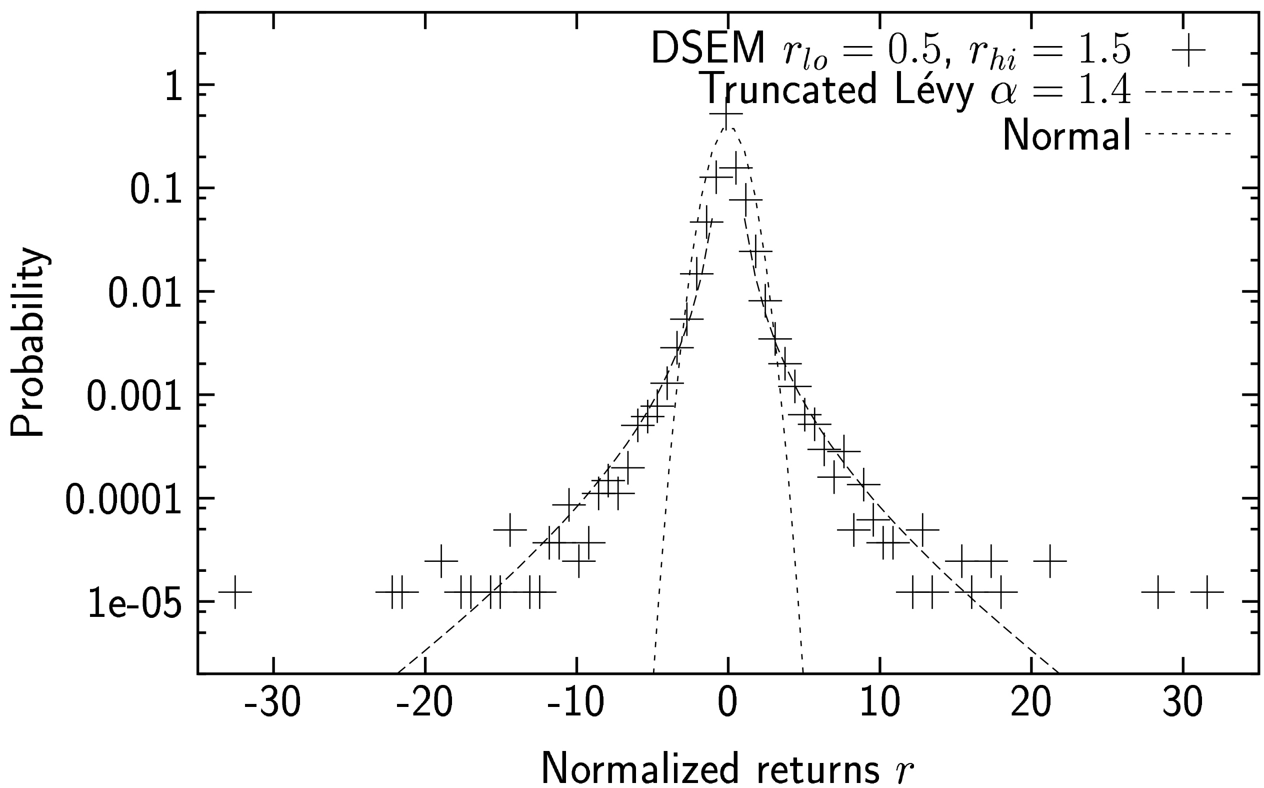
$$1 - H(r_p) \propto (1 - r_p)^b \tag{6}$$

gives exponent $b = 0.19 \pm 0.02$.

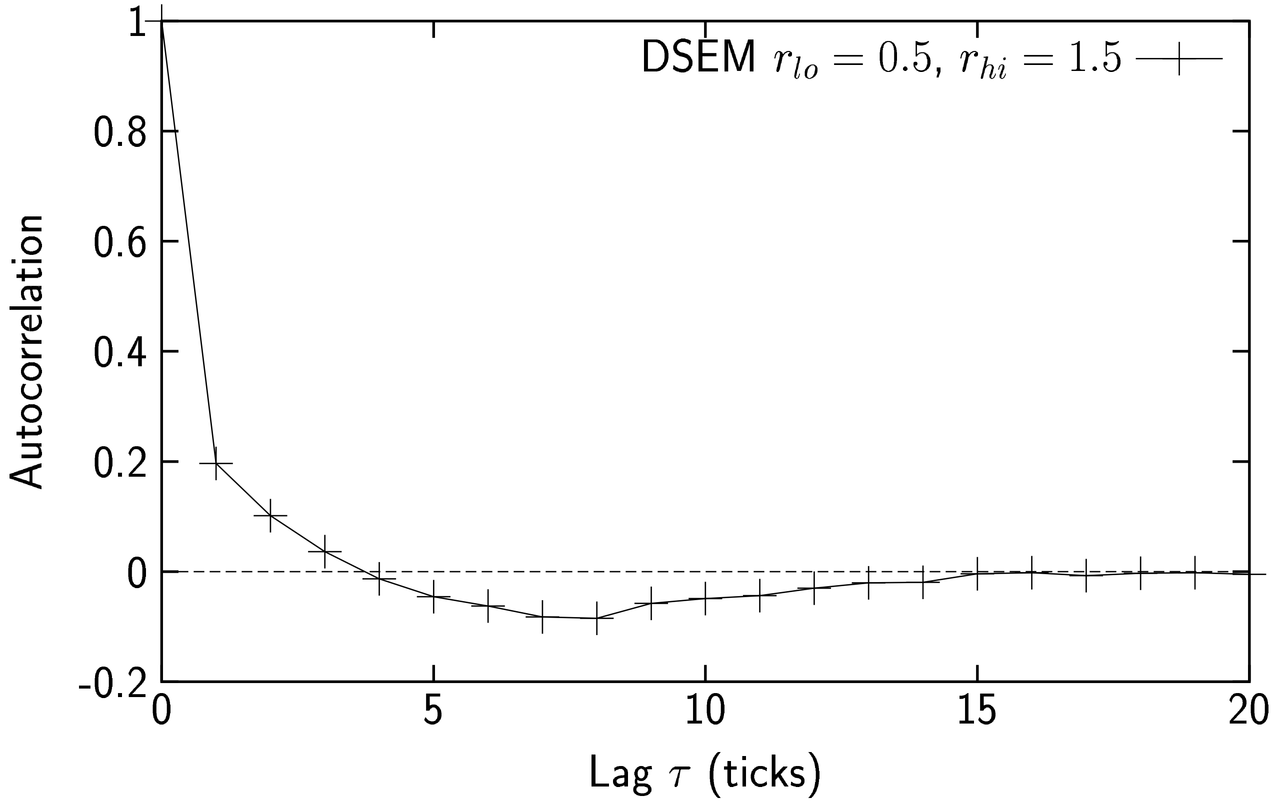
5 How well does it describe real markets?

When control parameter r_p spanned critical point, saw...

Fat tails:

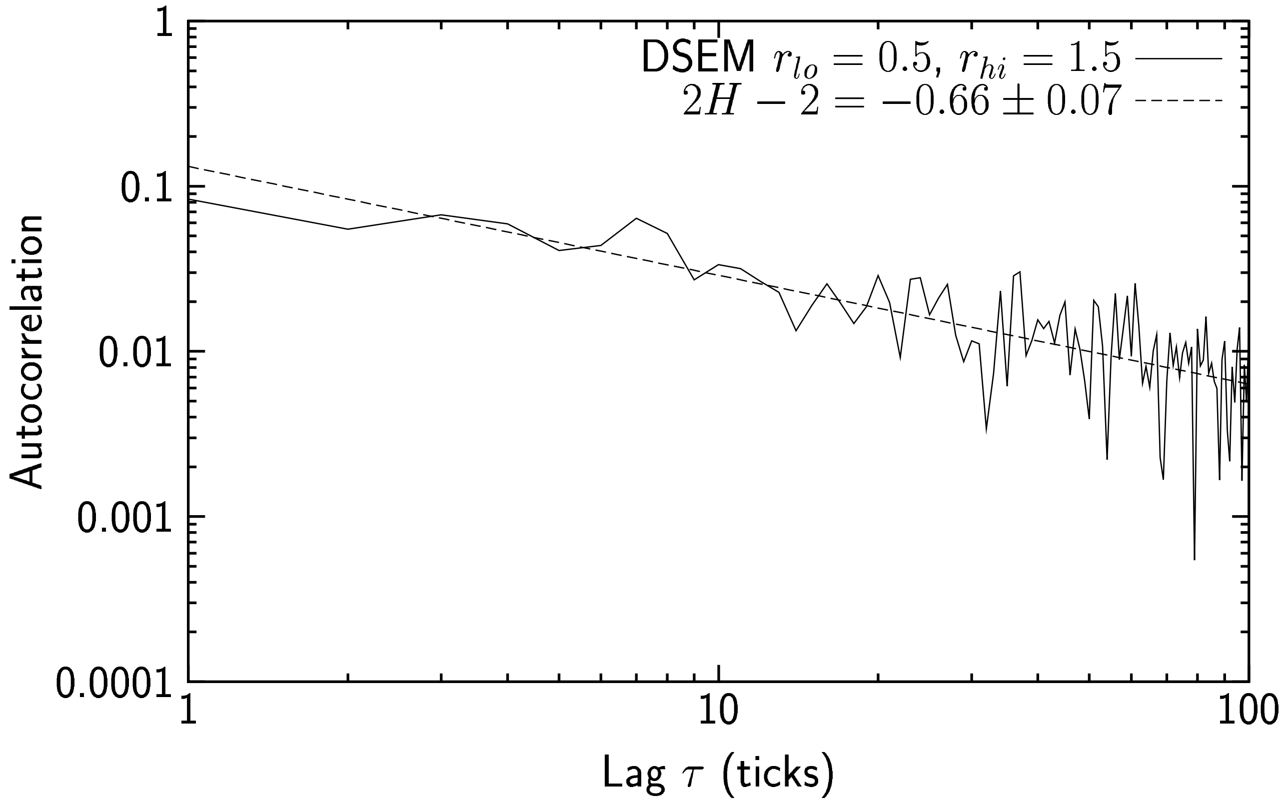


Short memory:



Note timescale is "ticks"—number of trades—not days.

Clustered volatility:



Hurst exponent is $H \approx 0.7$, lower than empirical $H \approx 0.9$ (but still significant).

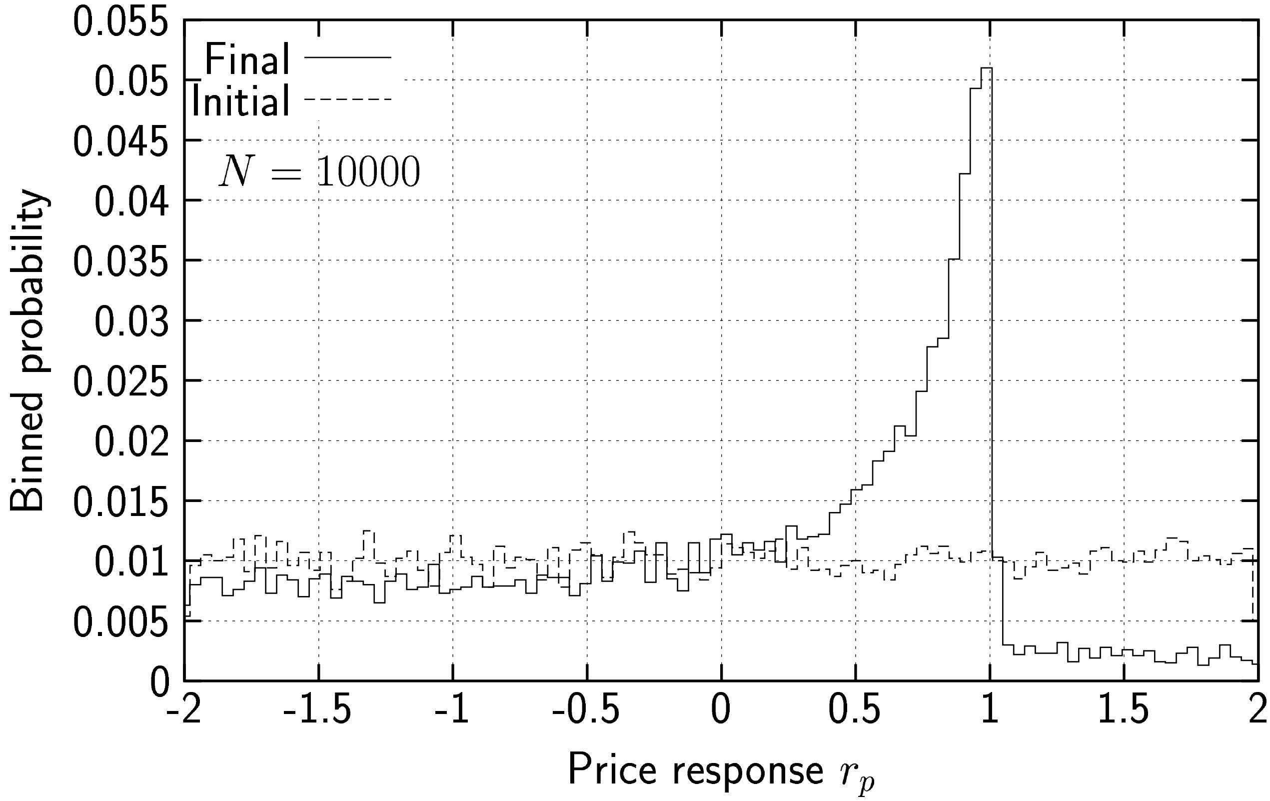
In conclusion, DSEM replicates empirical anomalies when it spans the critical point.

Does this mean that real markets operate at a critical point? If so, why?

6 What's new?

Since thesis, have considered adaptive agents: an agent who sells at a price lower than its average buying price will randomly choose new preferences (eg. price response).

Sample distribution of price response:



So the critical point $r_p = 1$ is preferred. Seems to self-organize to criticality.

7 Where can I find out more?

My thesis is available from http://rikblok.cjb.net/phd. Can also find links to simulations there.

Or check out these...

References

[1] B. B. Mandelbrot, J. Business **36**, 394 (1963).

[2] R. N. Mantegna and H. E. Stanley, Nature **376**, 46 (1995).

[3] Dow Jones Industrial Average: Daily close, 1896–1999, available from http://www.economagic.com/em-cgi/data.exe/djind/day-djiac, provided by Economagic.com.

[4] R. C. Merton, *Continuous-Time Finance* (Blackwell, Cambridge, 1992).

[5] E. T. Jaynes, http://bayes.wustl.edu/etj/prob.html (unpublished).

[6] P. Gopikrishnan *et al.*, Phys. Rev. E **60**, 5305 (1999), arXiv:cond-mat/9905305.

A Truncated Lévy flight

Best description of price returns is truncated Lévy flight $1/r^{\alpha+1}$.

Must be truncated because, empirically, variance is finite.

Some debate over how it is truncated.

Best analysis thus far [6] indicates tails crossover to power law with $\alpha = 3$.

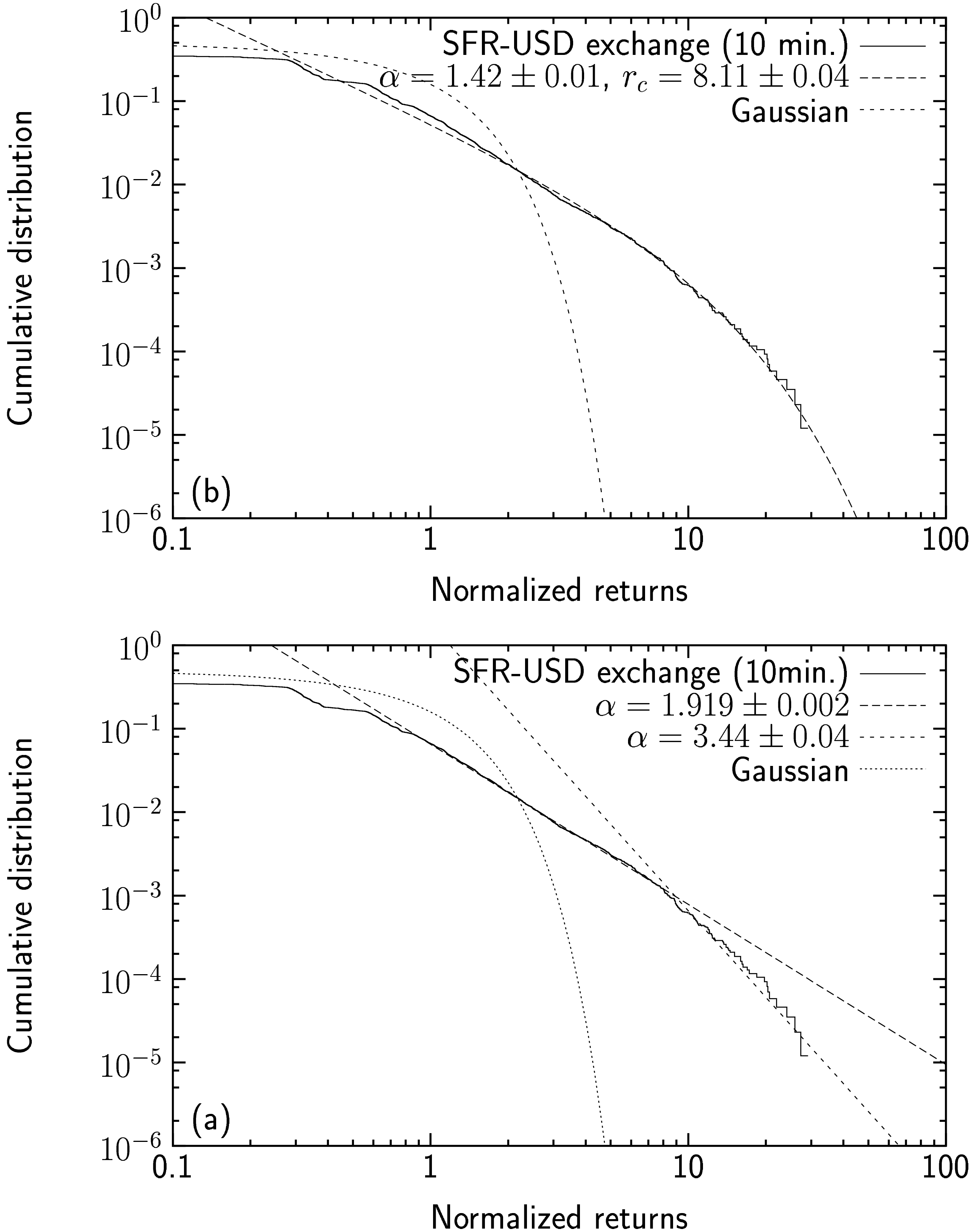
But I assumed exponential truncation. Cumulative distribution obeys

$$C(r \to \infty) \propto r^{-\alpha} \exp(-r/r_c). \tag{7}$$

Preferred because: (1) only requires three parameters (versus five), and (2) is linear in parameters.

Can be tested which method is more suitable because inverse cubic predicts divergent kurtosis (verified empirically).

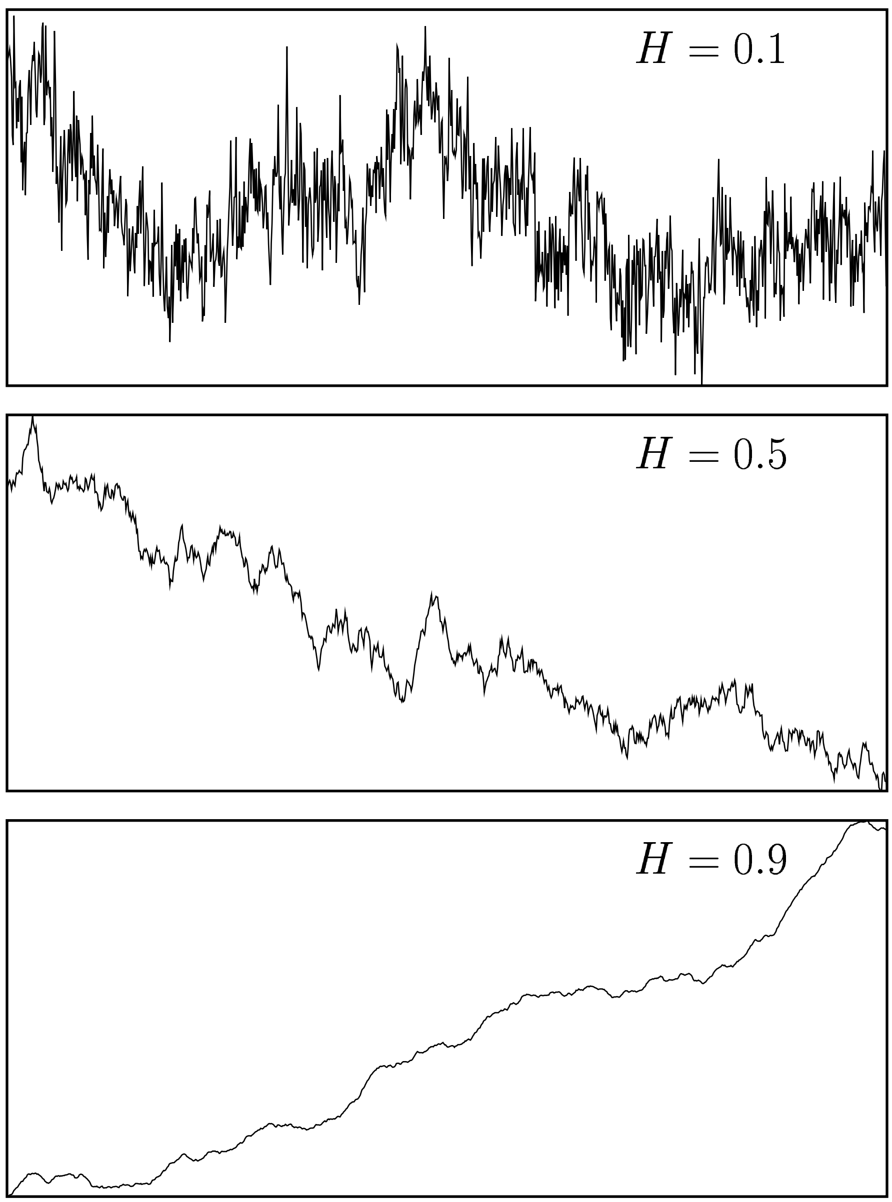
But in many cases exponential truncation is "good enough"...


SFR-USD exchange (10 min.)
$\alpha = 1.42 \pm 0.01$, $r_c = 8.11 \pm 0.04$
Gaussian
Cumulative distribution
Normalized returns
(b)
SFR-USD exchange (10min.)
$\alpha = 1.919 \pm 0.002$
$\alpha = 3.44 \pm 0.04$
Gaussian
Cumulative distribution
Normalized returns
(a)

B Hurst exponent

Fractional Brownian motion extension of Brownian motion to standard deviation which grows as $\sigma(t) \propto t^H$.

$H < 1/2$ anticorrelated, $H = 1/2$ uncorrelated, $H > 1/2$ correlated.



Measure H by analyzing fluctuations on different timescales. Eg. power spectrum: