Extra! Extra! Critical Update on ‘Life’

by
Rik Blok

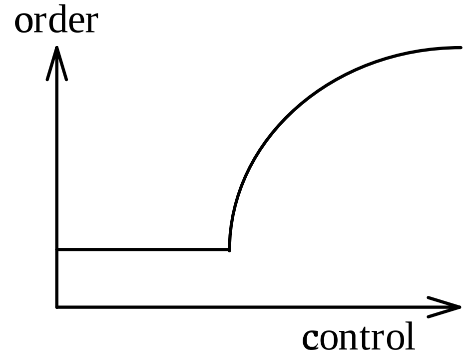
for
PWIAS Crisis Points

March 18, 1998

Self-organized Criticality (SOC)

***critical point*:**

- under variation of a *control parameter*, an *order parameter* undergoes continuous change with discontinuous derivative



- many suitable choices for order parameter
- behaviour near critical point governed by power-laws, many divergent properties
- dominated by fluctuations
- no characteristic scale, can have events/structures on *all* scales
- *universality*: many details of the system are irrelevant to the critical behaviour, systems sharing the same few important properties (eg. dimensionality) belong to the same *universality class*

+

***self-organization*:**

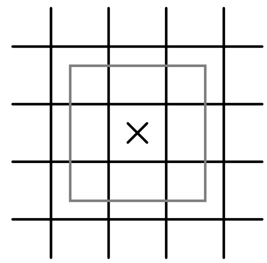
- the system spontaneously approaches the critical point without tuning a *control parameter*

= SOC [1,2,3,4]

Game of Life

"Game of Life simulates the evolution of a colony of organisms and suggests that the theory of self-organized criticality can explain the dynamics of ecosystems." [4]

- cellular automaton
- 2D square lattice
- each site has 2 possible states, *alive* or *dead*
- 8 nearest neighbors
- discrete time, all sites updated in parallel



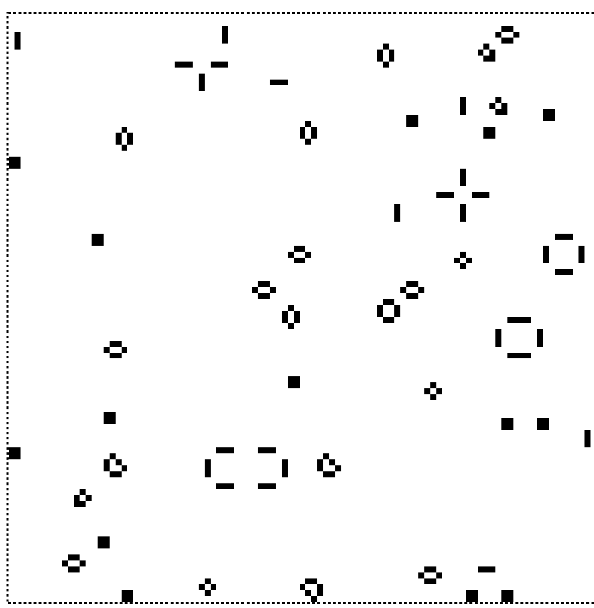
Rules

- Births: *dead* and exactly three (*live*) neighbors at time $t \rightarrow$ *alive* at t+1
- Survivals: *alive* and either two or three neighbors at time $t \rightarrow$ *alive* at t+1
- Deaths: neither above condition at $t \rightarrow$ *dead* at t+1

or

$$c_{t+1} = (1 - c_t)\delta_{n,3} + c_t(\delta_{n,2} + \delta_{n,3})$$

where c is state of cell (0=*dead*, 1=*alive*), n is count of live neighbors, and δ is the Kronecker delta function (returns 1 if the indices are equal, else zero)



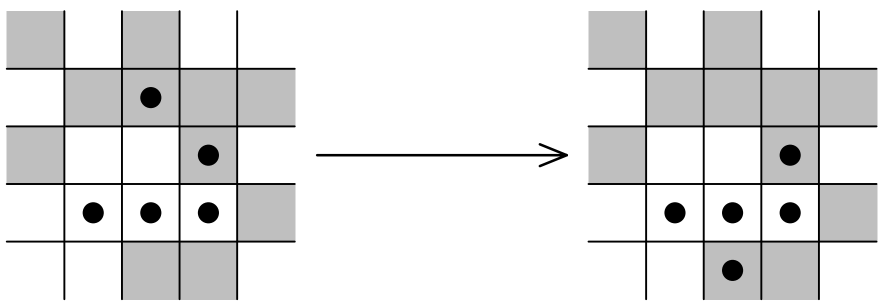
Sample steady-state configuration.

Synchronicity

- complicate matters by adding another parameter, *s*
- instead of updating all sites simultaneously, each site has a probability ***s*** of being updated in each time step
- call ***s*** "synchronicity"
- can adjust *s* to continuously vary between parallel (s=1) and Poisson ("one-at-a-time", $s \to +0$) updating

Example

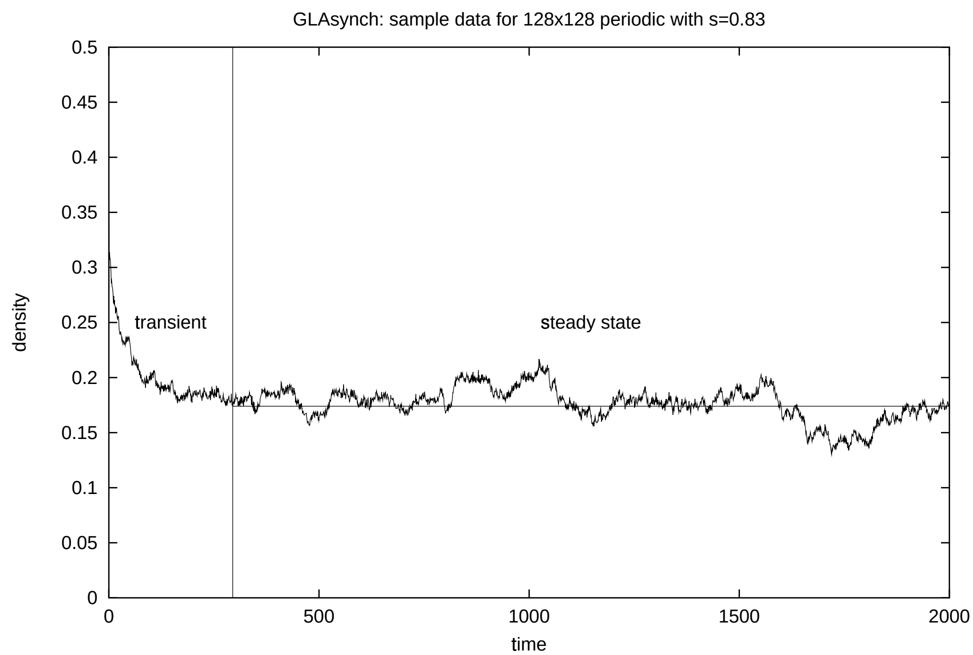
- ***s***= ½
- sites to be updated represented in gray

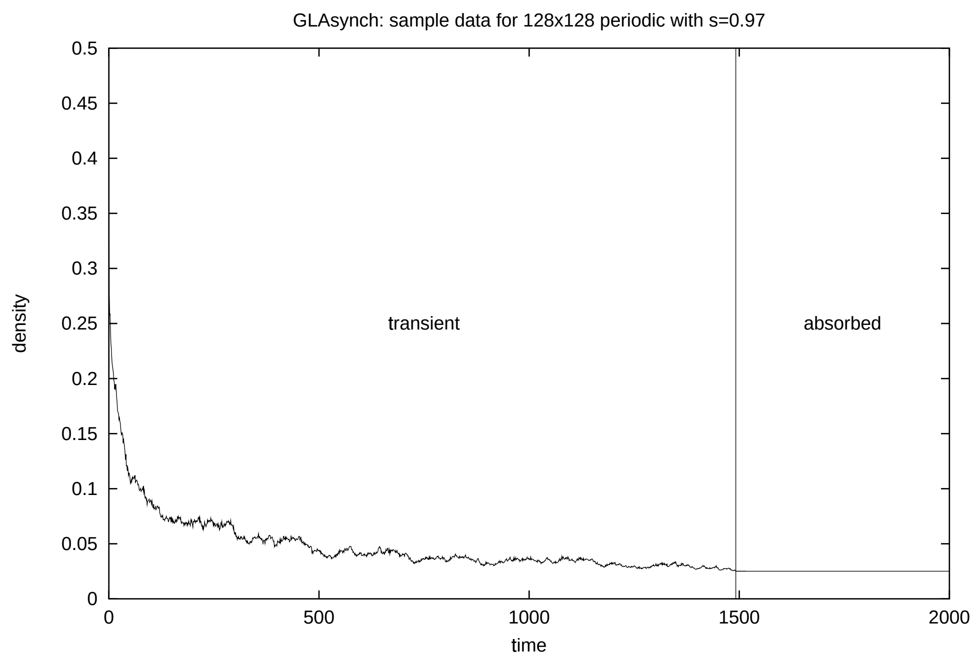


Analysis

Collect time-series of interesting statistics, such as:

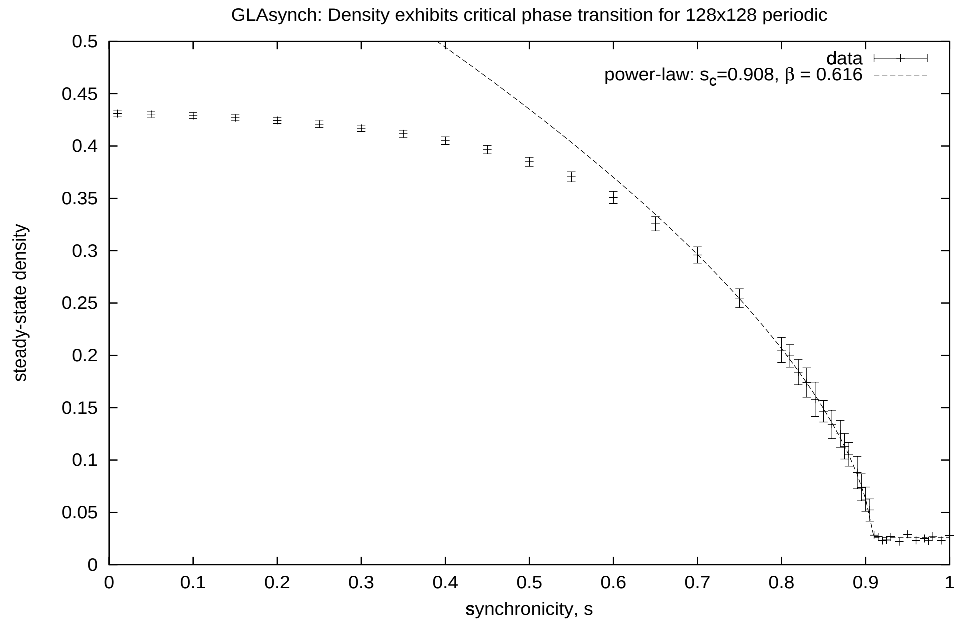
- density, ρ ≡ fraction of *live* sites (eg. ρ = 5/25 in above)
- activity, ***a*** ≡ fraction of updated sites which have changed state (births and deaths, eg. ***a*** = 2/11 in above)

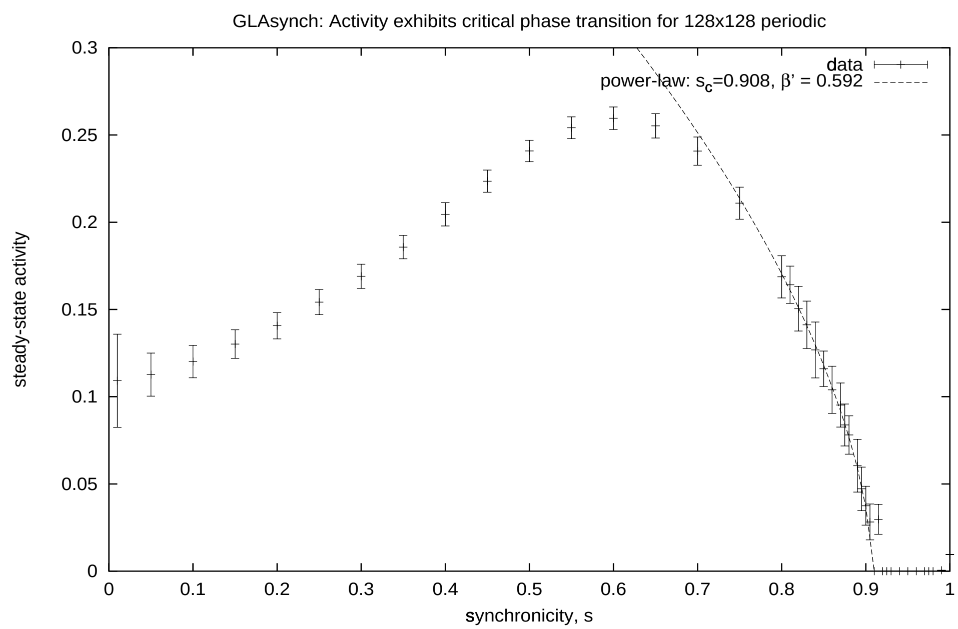




- time is rescaled by $\boldsymbol{s}$ ($t \rightarrow st$)
- qualitatively different evolutions suggest a phase transition

Critical Point

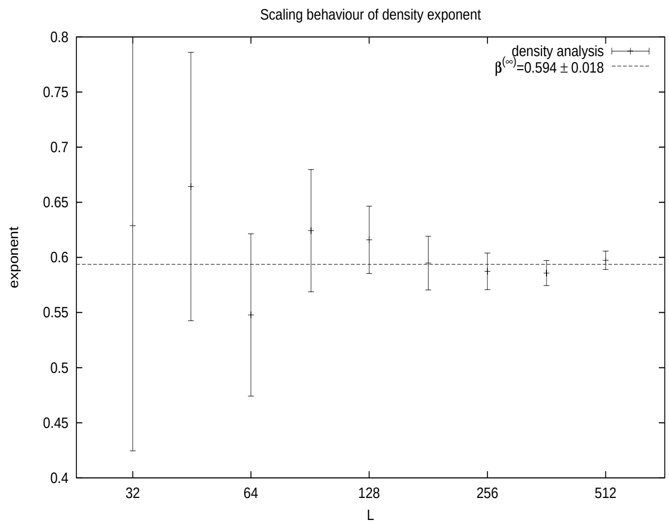


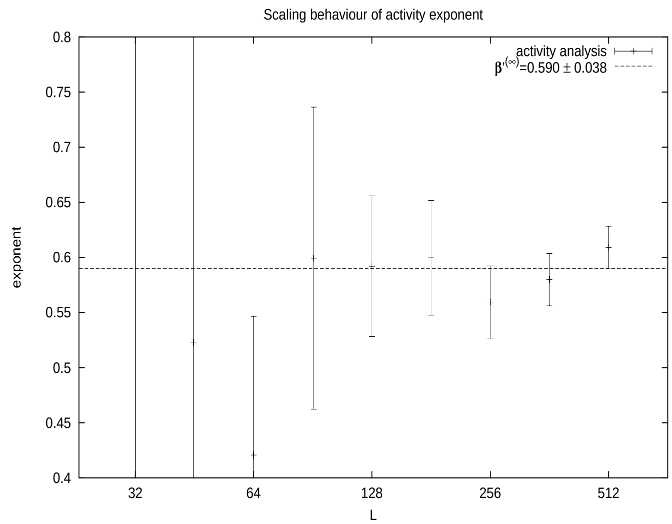


- fit to $y - y_0 \propto (s_c - s)^{\beta}$
- recovered exponent and critical point:
  - density: $s_c = 0.908 \pm 0.006$, $\beta = 0.616 \pm 0.031$, $y_0 \approx 0.0257$
  - activity: $s_c = 0.908 \pm 0.025$, $\beta' = 0.592 \pm 0.064$, $y_0 = 0$
- so GL is “close to” a dynamical critical point, which makes it *look* SOC

Scaling

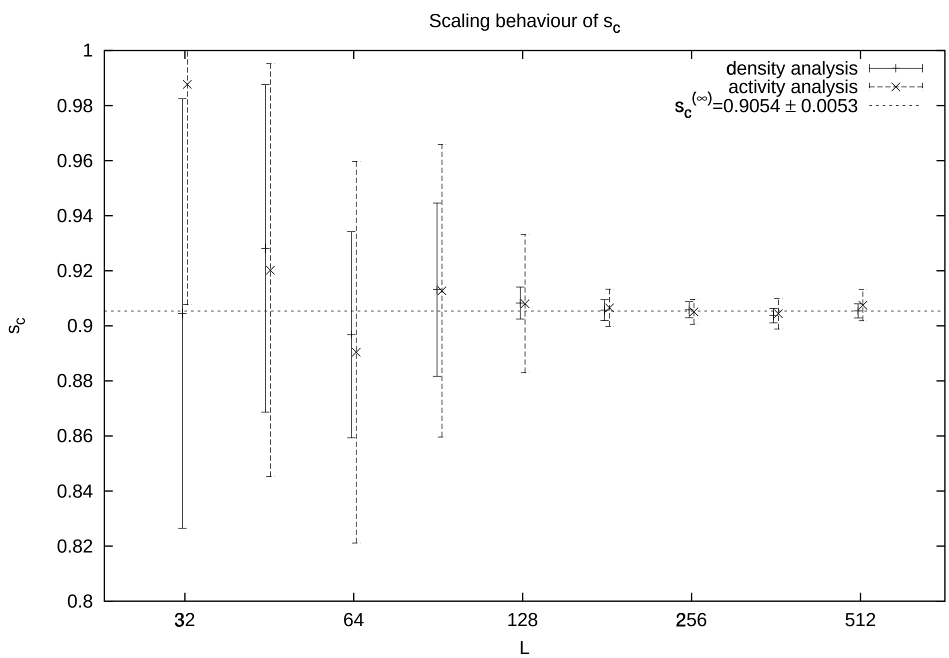
- runs on $L \times L$ lattices with periodic boundaries
- want to minimize boundary effects





$\beta^{(\infty)} = 0.594 \pm 0.018$

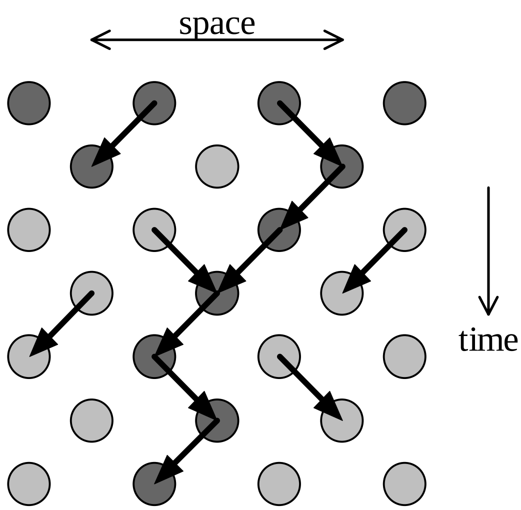
$\beta'^{(\infty)} = 0.590 \pm 0.038$



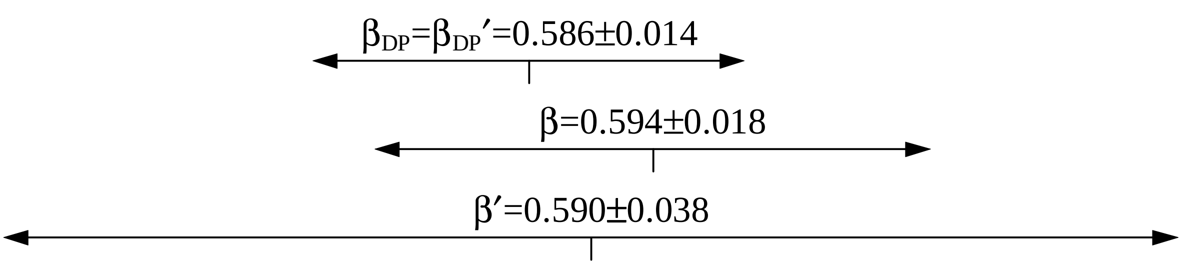
$s_c^{(\infty)} = 0.9054 \pm 0.0053$

Directed Percolation

- uniform lattice of sites
- bonds between neighbors are randomly distributed with concentration ***p***
- bonds are restricted to a certain direction (often related to ***time***)



- critical point, p_c, above which there is a nonzero probability of propagation to infinity
- critical behaviour independent of details like lattice structure
- in 2+1 dimensions $\beta_{DP}=\beta_{DP}'=0.586\pm0.014$ [5]



- GL(s_c) appears to be in same universality class as DP
- class also includes
    - Reggeon field theory (high energy physics) [6]
    - surface reaction models [5]
    - contact process [7 and *Durrett*, *1988*]
    - Domany-Kinzel cellular automaton [8]

Conclusions

- updating (parallel or sequential) important
- GL looks SOC because near critical point
- SOC is just criticality with an undiscovered control parameter [9]
- appears to belong to DP universality class

References

[1] Bak P, Tang C and Wiesenfeld K (1987) *Phys. Rev. Lett.* **59** 381-4

[2] Bak P, Tang C and Wiesenfeld K (1988) *Phys. Rev. A* **38** 364-74

[3] Bak P, Chen K and Creutz M (1989) *Nature* **342** 780-2

[4] Bak P and Chen K (1991) *Sci. Am.* **264** (1) 46-53

[5] Grinstein G, Lai Z-W and Browne D A (1989) *Phys. Rev. A* **40** 4820-3

[6] Deutscher G, Zallen R and Adler J (1983) *Percolation Structures and Processes* (Adam Hilger, The Israel Physical Society and The American Institute of Physics)

[7] Jensen I and Dickman R (1993) *Phys. Rev. E* **48** 1710-25

[8] Dickman R and Tretyakov A Y (1995) *Phys. Rev. E* **52** 3218-20

...many more on DP...

[9] Sornette D, Johansen A and Dornic I (1995) *preprint* (to appear in *J. de Physique I* **5**)