

**ON THE NATURE OF THE STOCK MARKET:  
SIMULATIONS AND EXPERIMENTS**

by

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## Abstract

Over the last few years there has been a surge of activity within the physics community in the emerging field of *Econophysics*—the study of economic systems from a physicist’s perspective. Physicists tend to take a different view than economists and other social scientists, being interested in such topics as phase transitions and fluctuations.

In this dissertation two simple models of stock exchange are developed and simulated numerically. The first is characterized by centralized trading with a market maker. Fluctuations are driven by a stochastic component in the agents’ forecasts. As the scale of the fluctuations is varied a critical phase transition is discovered. Unfortunately, this model is unable to generate realistic market dynamics.

The second model discards the requirement of centralized trading. In this case the stochastic driving force is Gaussian-distributed “news events” which are public knowledge. Under variation of the control parameter the model exhibits two phase transitions: both a first- and a second-order (critical).

The decentralized model is able to capture many of the interesting properties observed in empirical markets such as fat tails in the distribution of returns, a brief memory in the return series, and long-range correlations in volatility. Significantly, these properties only emerge when the parameters are tuned such that the model spans the critical point. This suggests that real markets may operate at or near a critical point, but is unable to explain why this should be. This remains an interesting open question worth further investigation.

One of the main points of the thesis is that these empirical phenomena are not present in the stochastic driving force, but emerge endogenously from interactions between agents. Further, they emerge despite the simplicity of the modeled agents; suggesting complex market dynamics do not arise from the complexity of individual investors but simply from interactions between (even simple) investors.

Although the emphasis of this thesis is on the extent to which multi-agent models can produce complex dynamics, some attempt is also made to relate this work with empirical data. Firstly, the trading strategy applied by the agents in the

second model is demonstrated to be adequate, if not optimal, and to have some surprising consequences.

Secondly, the claim put forth by Sornette *et al.* [1] that large financial crashes may be heralded by accelerating precursory oscillations is also tested. It is shown that there is weak evidence for the existence of *log-periodic precursors* but the signal is probably too indistinct to allow for reliable predictions.

# Contents

<b>Abstract</b>	<b>ii</b>
<b>List of Tables</b>	<b>ix</b>
<b>List of Figures</b>	<b>xi</b>
<b>Acknowledgements</b>	<b>xix</b>
<b>Chapter 1 Introduction</b>	<b>1</b>
1.1 Financial markets . . . . .	1
1.2 Motivation for research . . . . .	1
1.2.1 Motivation for the physicist . . . . .	2
1.3 Anticipated challenges . . . . .	5
1.4 Modeling . . . . .	6
1.4.1 Computer simulations . . . . .	7
1.4.2 An appeal for simplicity . . . . .	8
1.5 Organization of the thesis . . . . .	8
<b>Chapter 2 Centralized Stock Exchange Model</b>	<b>10</b>
2.1 Inspiration . . . . .	10
2.2 Theory . . . . .	10
2.2.1 Assumptions . . . . .	11
2.2.2 Utility theory . . . . .	12
2.2.3 Optimal holdings . . . . .	14
2.2.4 Risk aversion . . . . .	15
2.2.5 Optimal investment fraction . . . . .	16
2.2.6 Forecasting . . . . .	17
2.2.7 Fluctuations . . . . .	19
2.2.8 Initialization . . . . .	20
2.2.9 Market clearing . . . . .	21

2.2.10	Review . . . . .	23
2.3	Implementation . . . . .	25
2.3.1	Pseudo-random numbers . . . . .	27
2.4	Parameter space exploration . . . . .	28
2.4.1	Number of agents $N$ . . . . .	28
2.4.2	Total cash $C$ and total shares $S$ . . . . .	28
2.4.3	Investment fraction limit $\delta$ . . . . .	31
2.4.4	Risk aversion $a$ and forecast uncertainty $\sigma_\epsilon$ . . . . .	31
2.5	Parameter tuning . . . . .	34
2.5.1	Forecast error . . . . .	37
2.5.2	Finalized parameter ranges . . . . .	40
2.6	Discussion . . . . .	40
2.6.1	Fundamentalists versus noise traders . . . . .	40
2.6.2	Forecasting . . . . .	40
2.6.3	Portfolios . . . . .	41
2.6.4	Difficulties . . . . .	41
<b>Chapter 3 Decentralized Stock Exchange Model</b>		<b>44</b>
3.1	Inspiration . . . . .	44
3.2	Basic theory . . . . .	45
3.2.1	Assumptions . . . . .	45
3.2.2	Utility theory . . . . .	47
3.2.3	Optimal investment fraction . . . . .	47
3.2.4	Fixed investment strategy . . . . .	48
3.2.5	Friction . . . . .	51
3.2.6	Call orders . . . . .	51
3.2.7	Poisson processes . . . . .	53
3.2.8	Call interval . . . . .	53
3.2.9	Reply orders . . . . .	56
3.2.10	Time scale . . . . .	57
3.2.11	Initialization . . . . .	58
3.3	Fluctuation theory . . . . .	58
3.3.1	Bayes' theorem . . . . .	58
3.3.2	News . . . . .	59
3.3.3	Price response . . . . .	61
3.3.4	Review . . . . .	63
3.4	Implementation . . . . .	65
3.5	Parameter space exploration . . . . .	65
3.5.1	Number of agents $N$ . . . . .	65

3.5.2	Total cash $C$ and total shares $S$ . . . . .	65
3.5.3	Further scaling . . . . .	68
3.6	Parameter tuning . . . . .	69
3.6.1	News response . . . . .	69
3.6.2	Friction . . . . .	71
3.6.3	News interval . . . . .	72
3.6.4	Finalized parameter ranges . . . . .	72
<b>Chapter 4 Analysis and Results: Phase space</b>		<b>74</b>
4.1	CSEM phase space . . . . .	74
4.1.1	Review . . . . .	74
4.1.2	Data collection . . . . .	74
4.1.3	Phases . . . . .	75
4.1.4	Investment limit . . . . .	77
4.1.5	Critical regime . . . . .	79
4.1.6	Alternative thermodynamic variables . . . . .	80
4.1.7	Finite size effects . . . . .	82
4.1.8	Transient . . . . .	87
4.1.9	Summary . . . . .	88
4.2	DSEM phase space . . . . .	90
4.2.1	Review . . . . .	90
4.2.2	Data collection . . . . .	92
4.2.3	Phases . . . . .	92
4.2.4	Phase transition to $H = 1$ at $r_p = r_1$ . . . . .	96
4.2.5	Phase transition to $H = 0$ at $r_p = r_2$ . . . . .	97
4.2.6	Summary . . . . .	101
4.3	Number of investors . . . . .	101
4.3.1	Centralized Stock Exchange Model . . . . .	103
4.3.2	Decentralized Stock Exchange Model . . . . .	106
4.3.3	Summary . . . . .	106
<b>Chapter 5 Analysis and Results: Empirical results</b>		<b>108</b>
5.1	Price fluctuations . . . . .	108
5.1.1	Background . . . . .	108
5.1.2	Alternative: Decaying power law . . . . .	110
5.1.3	Methodology . . . . .	111
5.1.4	Centralized stock exchange model . . . . .	114
5.1.5	Decentralized stock exchange model . . . . .	116
5.1.6	Summary . . . . .	122

5.2	Price autocorrelation . . . . .	125
5.2.1	Background: The efficient market hypothesis . . . . .	125
5.2.2	News . . . . .	125
5.2.3	Short timescales . . . . .	126
5.2.4	Long timescales . . . . .	127
5.3	Volatility clustering . . . . .	132
5.3.1	Shuffling . . . . .	133
5.4	Scaling and Clustered volatility . . . . .	134
5.5	Wealth distribution . . . . .	135
5.5.1	Challenges . . . . .	135
5.5.2	Log-normal distribution . . . . .	135
5.5.3	Two-point price response . . . . .	136
5.6	Summary . . . . .	138
<b>Chapter 6 Experiments with a hypothetical portfolio</b>		<b>140</b>
6.1	Motivation . . . . .	140
6.2	Choice of companies . . . . .	141
6.3	Friction . . . . .	142
6.3.1	Minimum friction . . . . .	142
6.4	FIS Experimental results . . . . .	144
6.4.1	Events . . . . .	144
6.4.2	Performance . . . . .	145
6.5	Log-periodic precursors . . . . .	150
6.5.1	Scale invariance . . . . .	150
6.5.2	Discrete scale invariance and complex exponents . . . . .	150
6.5.3	Log-periodic precursors . . . . .	151
6.5.4	Critical points . . . . .	151
6.5.5	Application to financial time series . . . . .	152
6.5.6	Experimental design . . . . .	153
6.5.7	Results . . . . .	155
6.5.8	Universality of scaling ratio . . . . .	158
6.6	Summary . . . . .	158
<b>Chapter 7 Concluding remarks</b>		<b>160</b>
7.1	Review . . . . .	160
7.1.1	Anomalous market properties . . . . .	160
7.1.2	Centralized stock exchange model . . . . .	161
7.1.3	Decentralized stock exchange model . . . . .	161
7.1.4	Fixed investment strategy . . . . .	162

7.1.5	Log-periodic precursors . . . . .	163
7.2	Conclusions to be drawn from this research . . . . .	163
7.3	Relation to other work in the field . . . . .	164
7.4	Avenues for further work . . . . .	165
<b>Bibliography</b>		<b>167</b>
<b>Appendix A Discounted least-squares curve fitting</b>		<b>177</b>
A.1	Least-squares curve fitting . . . . .	177
A.2	Discounting . . . . .	178
A.3	Storage and updating . . . . .	180
A.4	Memory . . . . .	182
A.5	Unknown measurement errors . . . . .	183
A.6	Forecasting . . . . .	183
A.6.1	Unknown measurement errors . . . . .	184
A.7	Summary . . . . .	185
<b>Appendix B Sampling discrete processes</b>		<b>186</b>
B.1	Simple random walk . . . . .	186
B.2	Poisson Brownian motion . . . . .	190
B.3	Sampling . . . . .	191
<b>Appendix C Long-range memory: The Hurst exponent</b>		<b>193</b>
C.1	Synthesis . . . . .	197
C.2	Analysis . . . . .	199
C.2.1	Dispersional analysis . . . . .	200
C.2.2	Scaled Window Variance analysis . . . . .	201
C.2.3	Lévy Flight . . . . .	202
C.3	Conclusions . . . . .	205



# List of Tables

2.1	All parameters and variables used in the Centralized Stock Exchange Model (CSEM). . . . .	26
2.2	Parameter values for CSEM Runs 1, 2 and 3. . . . .	29
2.3	Parameter values for CSEM Runs 4 and 5. . . . .	32
2.4	Parameter values for CSEM Run 6. . . . .	35
2.5	Regression analysis of $\log w$ versus agent parameters for different samples of Run 6 (Table 2.4). The symbols indicate the sign of the regression-line slope, or zero if it is insignificant (relative to its standard error). The results indicate that $a$ is positively correlated with wealth but $\sigma_\epsilon$ , $M$ and $d$ are largely irrelevant. . . . .	36
2.6	Parameter values for CSEM Run 7. . . . .	37
2.7	As Table 2.1 except with updated parameter ranges. These ranges will be used in subsequent simulations. . . . .	39
3.1	All parameters and variables used in the Decentralized Stock Exchange Model (DSEM). . . . .	64
3.2	Parameter values for DSEM Runs 1, 2 and 3. . . . .	66
3.3	As Table 3.1 except with updated parameter ranges. These ranges will be used in subsequent simulations. All parameters except $N$ and $r_p$ are firm. . . . .	73
4.1	Parameter values for CSEM Dataset 1. Some of the parameters were established in Chapter 2 and are common to all the runs. Dataset 1 explores two dimensions of phase space: $N$ and $\sigma_\epsilon$ . . . . .	75
4.2	Parameter values for CSEM Dataset 2. These runs are a variation of Dataset 1 (all unspecified parameters are duplicated from Table 4.1, $N = 100$ ) exploring a range of investment limits $\delta$ . . . . .	77
4.3	The threshold values of $\sigma_\epsilon$ separating the two phases of CSEM shown in Fig. 4.3 do not appear to depend on the investment limit $\delta$ . . . . .	78

4.4	Parameter values for DSEM Dataset 1. Some of the parameters were established in Chapter 3 and are common to all the runs. Dataset 1 explores two dimensions of phase space: $N$ and $r_p$ . . . . .	91
4.5	Parameter values for DSEM Dataset 2. These runs are a variation of Dataset 1 (all unspecified parameters are duplicated from Table 4.4) exploring a few other intermediate system sizes. . . . .	98
4.6	Parameter values for CSEM Dataset 3. These runs are a variation of Dataset 1 (all unspecified parameters are duplicated from Table 4.1) with many agents $N = 10,000$ . . . . .	103
5.1	Parameter values for DSEM Dataset 3. These runs are a variation of Dataset 1 (all unspecified parameters are duplicated from Table 4.4) run out for longer times (roughly 80 years). Also notice that for $r_p = 0.99$ the news response was reduced by an order of magnitude to keep the price within reason. . . . .	116
5.2	Parameter values for DSEM Dataset 4. These runs are characterized by a two-point distribution of the price response. Each agent chooses $r_p = r_{lo}$ or $r_{hi}$ with equal probability. (All unspecified parameters are duplicated from Table 4.4.) . . . . .	119
5.3	Linear correlation analysis between said variable and the logarithm of the characteristic return from DSEM Dataset 4. The correlation is strongest with the upper limit of the price response $r_{hi}$ . . . . .	121
6.1	Initial holdings of a hypothetical portfolio on January 4, 1999. . . . .	141
6.2	Events relating to the hypothetical portfolio which occurred during the course of the experiment. . . . .	144
6.3	Final holdings of a hypothetical portfolio on May 12, 2000. . . . .	147
6.4	First four moments of the distribution of log-returns for each stock, the two trading strategies under review and the Nasdaq Composite Index. The skewness characterizes the asymmetry of the distribution and the kurtosis indicates the presence of outliers. The average skewness is not found to be significant but the kurtosis is. . . . .	148
6.5	Average values and standard deviations of the daily portfolio returns $(w_t/w_{t-1} - 1)$ for all data and separately for days a crash was forecasted and not forecasted. . . . .	157
6.6	Same as Table 6.5 except only including data up until the observed decline on April 14. . . . .	157

# List of Figures

1.1	Sample phase transitions. A first-order transition (left) is characterized by a discontinuity in the order parameter, while a second-order (critical, right) is discontinuous in the first derivative. . . . .	4
2.1	The exponential utility function defined in Eq. 2.4 is often applied in finance. The goal wealth parameter $w_{goal}$ implicitly sets the risk aversion. . . . .	14
2.2	Demonstration of forecasting via polynomial curve extrapolation. Shown are forecasts produced by a simple moving average and a linear trend. The linear trend is able to anticipate reversals in returns. . . . .	18
2.3	The expected initial trading price depends only on the risk aversion multiplied by the uncertainty of returns, $a\sigma_\epsilon$ . As the aversion or uncertainty increases the initial value of the stock drops. . . . .	24
2.4	Comparison of time evolutions of (a) price and (b) volume for Runs 1, 2 and 3 as defined in Table 2.2. The price scales as the ratio of cash to shares and the volume scales as the number of shares. (In both plots Run 2 is offset to improve readability.) . . . . .	30
2.5	Comparison of time evolutions of price for Runs 4 and 5 as defined in Table 2.3. The price is not perfectly invariant under rescalings which preserve the constant $a\sigma_\epsilon$ . . . . .	33
2.6	Price history generated by CSEM with parameters listed in Table 2.4 (Run 6). The price almost reaches its theoretical maximum of \$999 (see Eq. 2.44) before collapsing. The agent state variables were sampled at the times indicated. . . . .	35
2.7	Plot of agent wealth versus (a) risk aversion and (b) forecast error. The best fit lines have slopes $5.2 \pm 1.4$ (positive correlation) and $4.7 \pm 8.8$ (no correlation), respectively. . . . .	38

2.8	Price history generated by CSEM with parameters listed in Table 2.6 (Run 7). The series has the undesirable property that the price spends much of its history at or nearing its ceiling (\$999). . . . .	39
2.9	Plot of agent wealth versus (a) cash and (b) shares held showing that most agents hold extreme portfolios of maximum cash and minimum shares, or vice versa. It appears that the method of calculating the investment fraction in CSEM (Eq. 2.19) is too sensitive to fluctuations. (It should be acknowledged the plots are truncated since the lowest wealth actually extends down to $10^{-25}$ , an unrealistic quantity since real money is really discretized with a minimum resolution of one penny.) . . . . .	42
3.1	The fixed investment strategy specifies how many shares to trade at a price $p$ given an investment fraction $i$ and an ideal price $p^*$ (from Eq. 3.14). As the current price drops toward zero the fractional change in shares diverges. . . . .	50
3.2	Call orders $p_o$ are placed at either the current price $p$ or the limit prices $p_B$ or $p_S$ , whichever is better. The spread between the limits increases with friction $f$ . . . . .	52
3.3	“Buy” and “Sell” call orders are modeled as independent Poisson processes with price-dependent rates. As the last trading price increases, the probability of a “Sell” order being called becomes much more likely than a “Buy.” . . . . .	55
3.4	Comparison of time evolutions of (a) price and (b) volume for Runs 1, 2 and 3 as defined in Table 3.2. The price scales as the ratio of cash to shares and the volume scales as the number of shares. (Run 2 is offset to improve readability. The gaps in (b) denote periods of zero volume.) . . . . .	67
3.5	The price series generated by Run 1 is compared with the expected price generated by Eq. 3.70, showing rough agreement (though with systematic deviations). . . . .	70
4.1	The price series plots for CSEM with $N = 100$ agents and $\sigma_\epsilon = 0.10$ (a) and $\sigma_\epsilon = 0.05$ (b) indicate a change of character of the dynamics. . . . .	76
4.2	The highest price in any given simulation increases as the forecast error decreases until it reaches its theoretical limit, creating two separate phases for the dynamics. . . . .	78

4.3	The maximum price in CSEM has a limit which depends on the investment limit $\delta$ . However, the threshold value of $\sigma_c$ for which the limit is first reached does not appear to depend on $\delta$ . . . . .	79
4.4	The best fits of power laws to CSEM Dataset 2 (and $N = 100$ from Dataset 1) yield the critical points (a) and scaling exponents (b) shown. The lines represent the weighted averages of the best fit values. . . . .	81
4.5	The average price (a) and variance of fluctuations (b) also exhibit scaling near the critical point $\sigma_c = 0.12$ for the data from CSEM Dataset 2. The deviation from scaling observed near the critical point in (b) is due to the finite size of the system ( $N = 100$ ) as will be seen in Fig. 4.7. . . . .	83
4.6	The best fits of power laws to CSEM Dataset 1 yield the critical points (a) and scaling exponents (b) shown. A finite-size scaling analysis (neglecting $N = 50$ ) reveals information on how the critical point changes with increasing investor numbers (a). For reasons discussed in the text, the exponent for $N = 1000$ is dropped from the estimate of the scaling exponent (b). . . . .	84
4.7	The variance of the log-price largely collapses to a single curve when multiplied by the system size $N$ for CSEM Dataset 1. This curve diverges as the critical point is approached with an exponent $\gamma = 1.29 \pm 0.02$ calculated from the largest system $N = 1000$ . (The critical points were taken from Fig. 4.6(a).) . . . . .	86
4.8	Duration of the transient period in CSEM (Dataset 1) before the price series settles down to some steady-state value. The transient grows near the critical point $\sigma_c \approx 0.08$ . . . . .	89
4.9	The maximum transient in CSEM appears to scale with the system size with an exponent $1.5 \pm 0.1$ . . . . .	90
4.10	Sample price series for DSEM with $N = 100$ . Negative values of $r_p$ (a) produce an anticorrelated series while positive values (b) result in positive autocorrelations. . . . .	93
4.11	The Hurst exponent increases with $r_p$ in DSEM as expected but with two surprising phase transitions emerging at larger system sizes: one near $r_p \approx -0.4$ and the other near $r_p \approx 1$ . . . . .	95
4.12	The best fits of power laws to DSEM Dataset 1 yield the scaling exponents shown. The average exponent is $0.185 \pm 0.016$ . . . . .	96
4.13	Sample fits of first- and second-order phase transitions to $N = 500$ (a) and $N = 1000$ (b) near $r_2$ in DSEM show that the power-law fits better for small $N$ but the first-order prevails for larger systems. . . . .	99

4.14	As the system size $N$ increases the critical exponent $b$ tends to zero. The line represents a power-law fit $b \propto N^m$ giving an exponent $m = -0.25 \pm 0.04$ . . . . .	100
4.15	The price series (a) and daily volume (b) of DSEM with $N = 200$ and $r_p = -0.45$ is a good example of intermittency. The dynamics fluctuate between two phases. . . . .	102
4.16	The price series of CSEM for $\sigma_\epsilon = 0.01$ (a) appears unaffected by changing the number of agents $N$ . In particular, occasional semi-periodic fluctuations (b) are observed for all system sizes. . . . .	104
4.17	The price series of CSEM for $\sigma_\epsilon = 0.15$ exhibits smaller fluctuations and a lower mean as the system size increases. (The lower mean may simply be because the system has not reached a steady state yet.) . . . . .	105
4.18	In DSEM the price series does not get more regular as the system size is increased—in fact the fluctuation grow. This is especially true for $r_p = -0.75$ (a) but it is also indicated to a lesser degree at $r_p = 0.90$ (b). . . . .	107
5.1	Ten minute returns (86,000 data points) of the Swiss franc–U.S. dollar exchange rate [2] (negative tail) compared to power law with crossover to $\alpha \approx 3$ (a) and power law with exponential drop-off presented in this section (b). . . . .	112
5.2	Both tails of the cumulative distribution of daily (normalized) returns for the Nasdaq Composite index between October 1984 and Jun 2000 (4,000 data points) fit well to a decaying power law. The power law is truncated by two standard deviations in the positive tail but extends almost to four in the negative tail. . . . .	113
5.3	Scaling in the distribution of returns is only observed well below the critical point $\sigma_\epsilon \ll \sigma_c$ in CSEM as indicated by large values of the characteristic return $r_c$ . For small $\sigma_\epsilon$ scaling occurs in both tails for daily returns but only for negative returns in monthly returns. . . . .	115
5.4	For $\sigma_\epsilon = 0.03$ in CSEM ( $N = 1000$ ) the distribution of positive (monthly) returns (upper) almost converges to a Gaussian but still has a slightly heavy tail. The negative returns (lower), however, exhibit scaling for $r < r_c \approx 5.4$ with an exponent $\alpha \approx 1.1$ . . . . .	117
5.5	DSEM only begins to exhibit scaling, as measured by a characteristic return exceeding three standard deviations, for price responses well below the first-order transition $r_2 = -0.33$ and as the price response approaches the critical point $r_1 = 1$ . . . . .	118

5.6	The characteristic returns in DSEM with a two-point distribution of price responses ( $r_{lo}$ and $r_{hi}$ ) exceeds the required threshold of $r_c = 3$ when $r_{hi}$ is large (a). Neglecting the dependence on $r_{lo}$ (b) it becomes clear that the characteristic return grows exponentially with the upper limit $r_{hi}$ , crossing the threshold near $r_{hi} \approx 1$ . . . . .	120
5.7	The characteristic return $r_c$ (a) and scaling exponent $\alpha$ (b) for DSEM with $r_{lo} = 0.00$ and $r_{hi} = 1.25$ . The characteristic return grows as the sampling interval is shortened, but the scaling exponent $\alpha$ is fairly constant ( $1.55 \pm 0.11$ ). . . . .	123
5.8	Fitting the decaying power law to DSEM with a two-point price response using returns on individual trades (rather than per unit time, as in Fig. 5.6) shows scaling still occurs in the same region of parameter space. . . . .	124
5.9	Sample price series for DSEM Dataset 4 ( $r_{lo} = 0.5$ , $r_{hi} = 1.5$ ) showing the price roughly tracks the exponential of the cumulative news $e^\eta$ . The proportionality constant is estimated from the data. . . . .	126
5.10	The autocorrelation between daily returns for the Dow Jones Industrial Average [3] decays rapidly to zero with an estimated characteristic timescale $\tau_c = 0.4 \pm 0.2$ days. (Being less than the sampling interval, this estimate is not precise.) . . . . .	127
5.11	The autocorrelation between tickwise returns for DSEM (with a two-point price response distribution) decays rapidly to zero for all runs sampled. . . . .	128
5.12	Sample dispersion plot (see Section C.2.1) demonstrating the phenomenon of crossover in the Hurst exponent to $H \approx 1/2$ on long timescales for DSEM with a two-point price response distribution. . .	129
5.13	A reproduction of Fig. 5.12 except with regularly sampled returns at an “hourly” interval (instead of tickwise). Short timescale anticorrelations crossing over to uncorrelated returns at long timescales are still observed so the effect is not an artifact of sampling tickwise. . .	130
5.14	Both the crossover point, or memory, (a) and Hurst exponent for short timescales (b) indicate that memory effects are minimized when $r_{lo} \geq 0.25$ in DSEM with a two-point price response distribution. (The high values of the Hurst exponent for $r_{lo} > 0.5$ (b) do not cause problems because the memory is very short in this region (a).) . . .	131

5.15	The Hurst exponent of the absolute returns, which measures the degree of clustered volatility, is strictly greater than one half for all parameter combinations in DSEM. It is particularly high when the upper limit of the two-point distribution $r_{hi}$ is large or when the lower limit $r_{lo}$ is small. . . . .	133
5.16	DSEM Dataset 4 ( $N = 100$ agents) is able to capture three important properties observed empirically when $r_{lo} > 0.35$ and $r_{hi} > 1.25$ . The curves are contours from previous plots: (1) characteristic return $r_c = 3$ from Fig. 5.8 (solid line); (2) memory in return series = 100 from Fig. 5.14(a) (dashed line); and (3) Hurst exponent for the absolute returns $H = 0.6$ from Fig. 5.15 (dotted line). . . . .	134
5.17	Sample distribution of agents' wealth from DSEM Dataset 3 ( $N = 100$ , $r_p = -0.50$ ). There is insufficient data to distinguish between a normal and a log-normal distribution. . . . .	136
5.18	Sample distribution of agents' wealth from DSEM Dataset 4 ( $N = 100$ , $r_{lo} = 0$ , $r_{hi} = 1$ ). The log-normal curves are calculated from each sub-population, revealing a strongly bimodal nature. . . . .	137
5.19	In DSEM with a two-point price response the wealth of each of the sub-populations $w(r_p)$ depends strongly on the magnitude of the price response $ r_p $ . The population with the smallest absolute price response ( $r_{hi}$ to the left of zero and $r_{lo}$ to the right) consistently has more wealth as indicated by the ratio of wealth between the two sub-populations. . . . .	137
6.1	Historical wealth using FIS versus (a) the Buy-and-Hold strategy and (b) the Nasdaq Composite Index over the same interval (rescaled to be equal at the start of the experiment). . . . .	146
6.2	Histograms of log-returns of capital $r_{t+1} = \log(w_{t+1}/w_t)$ for both strategies. Notice BHS exhibits more large fluctuations (fatter tails) than FIS. . . . .	148
6.3	Sample fit of Eq. 6.29 to portfolio wealth on May 12, 2000. The best fit parameters indicate a crash is anticipated on or around $t_c =$ July 4, 2000. . . . .	155
6.4	Daily wealth returns $(w_t/w_{t-1} - 1)$ are shown along with the dates forecasted to crash in (a). The qualities of the curve fits corresponding to the forecasted crashes, which suggest the reliability of the predictions, are shown in (b). . . . .	156
A.1	Comparison of weightings using standard and discounted windows. . . . .	179



A.2	Discounted least-squares fitting has a computational storage advantage over moving windows of $N$ data points when $N > M^2 + M + 2$ where $M$ is the number of parameters to be fitted. . . . .	182
B.1	When a random walk is generated at some regular interval and sampled at another, $\Delta$ , the number of jumps between samples will vary.	187
B.2	The kurtosis is only zero at integer values of the sampling interval $\Delta$ and diverges as the sampling interval approaches zero. . . . .	188
B.3	The distribution of increments for the random walk appears to have fatter tails than a normal distribution with the same variance when sampled at intervals of $\Delta = 1.05$ . However, the tails still drop off as $e^{-x^2}$ . . . . .	189
B.4	Discrete Brownian motion with Poisson-distributed jump intervals has tails which fall off exponentially (with a decay constant of 0.72), instead of as $e^{-x^2}$ , when sampled at regular intervals ( $\Delta = 1$ ). . . . .	191
C.1	Sample fractional Brownian motion time series with different Hurst exponents: antipersistent $H = 0.1$ (top) has negative long-range correlations, uncorrelated $H = 0.5$ (center) is standard Brownian motion, and persistent $H = 0.9$ (bottom) has positive long-range correlations. . . . .	195
C.2	Power spectral densities for the fractional Brownian motion time series shown in Fig. C.1. The points are from finite samples of 1000 points each and the line represents the theoretical spectrum. For low frequencies the power spectrum is well approximated by a power law $1/f^{2H+1}$ . . . . .	196
C.3	Scaled window variance analyses for the fractional Brownian motion time series shown in Fig. C.1 (exact $H=0.1$ , $0.5$ , and $0.9$ , respectively). The estimated values of $H$ shown represent the best fit slopes of the lines. The analysis used $M_{min} = 4$ (see the text). . . . .	203
C.4	Comparison of Hurst estimators using synthetic datasets of 1000 points each. The scaled-window variance method (SWV, *) performs significantly better than rescaled range analysis ( $R/S$ , +) and marginally better than dispersional analysis (Disp., $\times$ ). (The points are offset slightly to improve readability.) . . . . .	204

C.5	Comparison of Hurst estimators on uncorrelated Lévy flight with characteristic exponent $\alpha$ using synthetic datasets of 1000 points each. Rescaled range ( $R/S, +$ ) and dispersional analysis (Disp., $\times$ ) perform well but scaled window variance analysis (SWV,*) performs poorly, especially for small $\alpha$ , tending towards the $1/\alpha$ curve. (The points are offset slightly to improve readability.) . . . . .	206
C.6	Schematic representation of relation between fractional Brownian motion and Lévy flight. Traditional Brownian motion sits at the intersection ( $H = 1/2, \alpha = 2$ ). The natural extension into the two-space is fractional Lévy motion which has correlated, non-Gaussian increments.	206

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