ISCI 344 Game Theory
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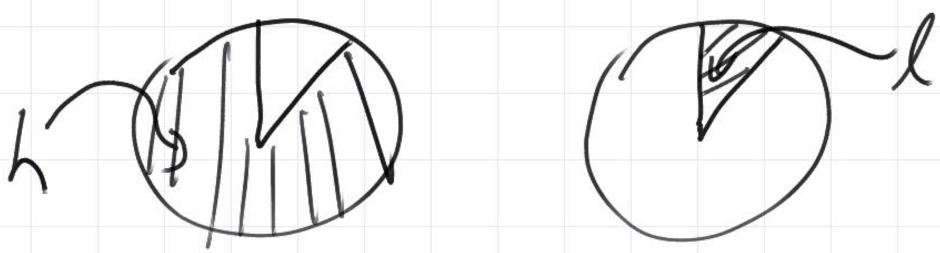
Lecture 1 – The Ultimatum Game

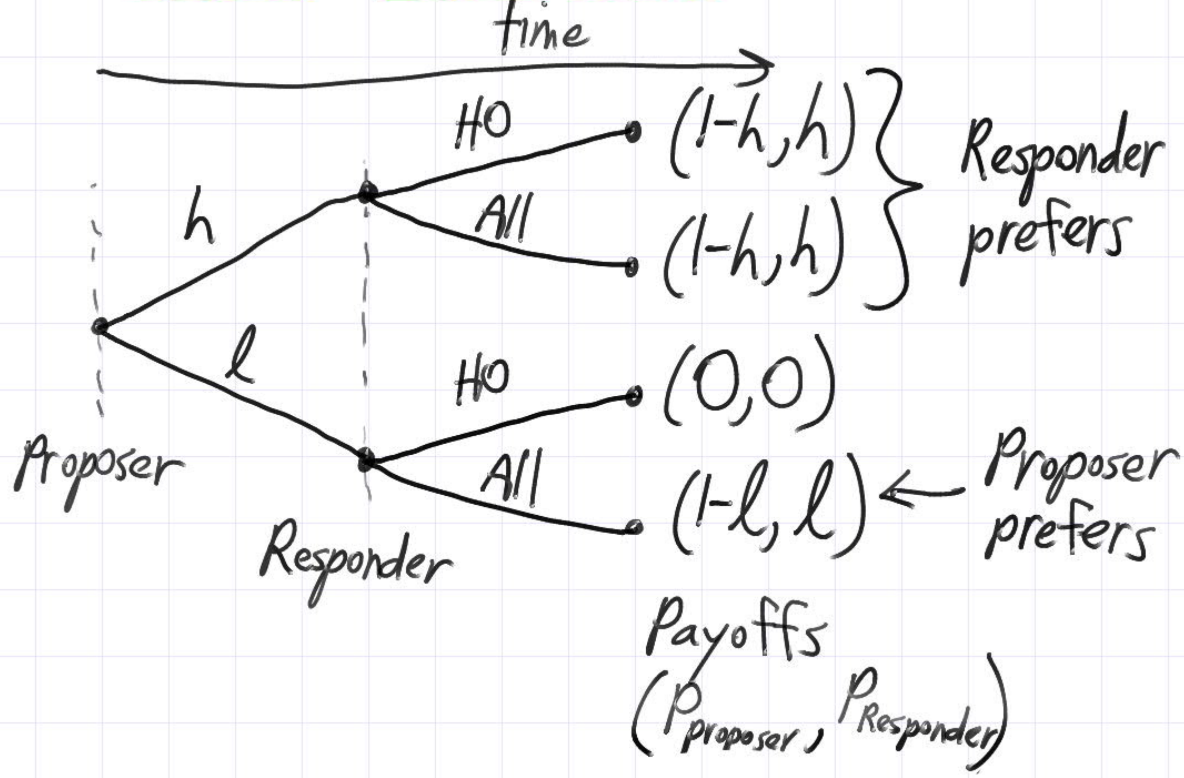
Ultimatum Game
- 2 players: Proposer & Responder
- have to agree how to share a payoff — a divisible commodity
- Proposer offers fraction to responder
- Responder can accept or decline
- if accepted then both players get their share
- if declined then neither gets anything
- Game Theory:
  - How much does Proposer offer?
  - When does Responder accept/decline?

Simplify: Proposer can offer: h = high fraction
or ℓ = low fraction
where $0 < \ell < h < 1$.



Simplify: Responder can: HO = accepts high offers only
All = accepts all offers
~~LO = low only~~
~~None = decline all offers~~

Decision tree — Extensive form



- Proposer & Responder prefer different outcomes — makes game interesting
- How do they decide?

Analysis: work backwards

1) Proposer makes high offer: both strategies of Responder yield identical outcomes → Responder has no reason to discriminate between strategies → Responder is indifferent (to choices)

2) Proposer makes low offer: Because $l > 0$ Responder prefers "accept All" strategy. Yields a higher payoff.

⇒ Responder's "accept All" strategy works in both cases.

3) Proposer does same analysis and concludes Responder will choose "accept All".

4) Given that knowledge, what does Proposer choose?
→ Proposer offers "low" because $1-l > 1-h$.

⇒ Expected outcome is $(1-l, l)$, ie. Proposer offers "low" and Responder "accepts All".

What are key assumptions in this analysis?

- higher payoffs are <u>always</u> preferred
  → payoffs accurately reflect preferences
  → in economics payoffs often termed <u>utilities</u> (we will discuss utilities more later)
  → players are <u>rational</u> — make choices that maximize payoffs (we will talk more about rationality)
- players do not care about other player's payoff — only their own payoff counts!
  → in general, it isn't even possible to compare payoffs between players!
  → if players do care about other player, then this would need to be reflected in a change to the original player's payoffs/utilities, to accurately reflect preferences

Aside: Chrissy cares as much for Rikky as for himself.
Rikky only cares about the difference in results*

If they get results* P_C & P_R

$$U_C = P_C + P_R$$
$$U_R = P_R - P_C$$

* We shouldn't call these payoffs. For consistency, let's define payoff = utility, <u>always</u>. So here, we should say Chrissy's utility (or *payoff*) is the sum of their results, and Rikky's is the difference.

2 key assumptions:
- higher payoffs always preferred
- only their own payoffs matter

→ foundational pillars of game theory (basis for all subsequent discussions throughout the course!)

What is special about the outcome $(1-\ell, \ell)$? If anything? (Under the above assumptions.)

- What happens if Proposer or Responder change strategy?
  - Proposer: $\ell \to h$: payoff drops, $1-\ell \to 1-h$
    → no incentive to switch
  - Responder: All → HO: payoff drops, $\ell \to 0$
    → no incentive to switch

⇒ no player can improve its payoff by <u>unilaterally</u> changing strategy

→ definition of "<u>Nash equilibrium</u>"

(John Nash, Nobel laureate)