UBC ISCI 344 Game Theory
Extensive and normal form games
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Outline:
- outcomes and utilities
- solution concepts
- extensive vs. normal form games

Outcomes and utilities:
- outcomes describe particular results of a game
  - → often written as set of strategies played or resulting payoffs
- utilities are numerical quantities that describe preferences players have for various outcomes
  - → may depend on numerous factors (eg. money/wealth, risk, fairness, reputation, hunger, ...)
  - → often money used as simple proxy for utilities (eg. experimental games)
  - → for convenience will assume that payoffs <u>are</u> utilities (unless explicitly stated)

Example: 3 outcomes, A B C

Player α utilities: 0 100 200

→ preferences: $A < B < C$ → prefers C

Player β utilities: 0 −1 +1

→ preferences: $B < A < C$ → prefers C

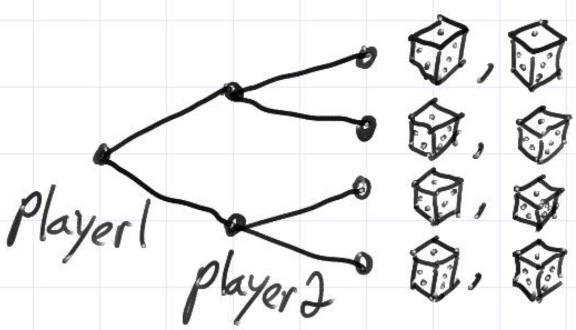
→ cannot conclude that player α has stronger preference for C than player β.

Solution concepts:

- Game Theory gives techniques to "solve" games — to predict likely results
- unlike some other math, no single "correct" solution
  - → solution concepts are techniques that highlight likely outcomes out of all possibilities
- examples:
  - subgame perfect equilibrium (SPE)
  - Nash equilibrium (NE)
  - Pareto optimum (PO)
  - many more ...
- once utilities are known solution concepts can be applied systematically — it no ~~no~~ longer matters where they came from

→ analysis solely based on outcomes/preferences

→ for exercises we can just put together trees/matrices with random payoffs, eg.



$\}$ $6^8 \sim 2$ million games

→ illustrates techniques but results have no implications because context is missing

⟹ key of project (and many discussions in class):
- Grasp real-world situation that can be represented as a game;
- Abstract scenario into formal game;
- Manipulate using solution concepts; and
- Explain implications for real-world scenario

Extensive form games (decision trees):
- analysis based on finding subgame perfect eq. (SPE)
- procedure: starting at end, eliminate all branches that are based on irrational decisions

→ SPE is unique if no player is indifferent at their decision node (eg. utilities of all outcomes for each player are different)

- there are other ways to analyze games

Normal form games (matrix games):
- example: Ultimatum game

(column) responder

(row) proposer	high only	all
high	$1-h$, h	$1-h$, h
low	0, 0	$1-\ell$, ℓ

column player

row player

- every extensive form game can be mapped onto a normal form
- normal form doesn't explicitly show that decisions are made sequentially (eg. Ultimatum game)
- often players have to decide without knowing opponent's decision (simultaneous game)
- we know that $(1-\ell, \ell)$ or (low, all) is SPE
  → what is special about this outcome (in matrix form) if anything?
    - proposer: low → high, payoff drops, $1-\ell \to 1-h$
      → no incentive to switch
    - responder: all → high only, payoff drops, $\ell \to 0$
      → no incentive to switch

⇒ no player can improve its payoff by unilaterally changing strategy!
= definition of Nash equilibrium (NE)

→ players cannot agree on simultaneous change of strategies (non-cooperative game theory — no discussion/no coalition formation among players) as opposed to cooperative game theory where this is possible. Here we focus on non-cooperative games.

- any other observations? Other NE?

		responder	
		high only	all
proposer	high	$1-h$, WNE h	$1-h$, h
	low	0, 0	$1-\ell$, SNE ℓ

"Preference arrows"
- $x \rightarrow y$: y preferred to x
- $\Leftrightarrow$: indifferent

SNE = strict NE, WNE = weak NE

→ (high, high only) is another NE but weak NE because responder is indifferent to switching
→ if responder can convince proposer that she will accept high only, then WNE expected

Summary:
- outcomes, utilities, payoffs, preferences
- solution concepts: SPE, NE + more
- Grasp, Abstract, Manipulate, Explain (GAME)
- extensive form (decision trees) and normal form (matrix) games
- sequential vs. simultaneous games
- cooperative vs. non-cooperative games
- strict vs. weak NE