UBC ISCI 344 Game Theory
Dominance and Pareto Optimality
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Outline:
- payoff matrices
- dominance
- strict vs. weak dominance
- iterated elimination of strictly dominated strats.
- Pareto improvement
- Pareto optimality

Payoff matrices:

- $M \times N$ payoff matrix: 2 players, row player has M strategies, column player has N

$$
\begin{array}{cc|cccc}
 & & & \text{Column} & & \\
 & & C_1 & C_2 & \cdots & C_N \\
\hline
 & R_1 & \square & \square & \cdots & \square \\
\text{Row} & R_2 & \square & \square & \cdots & \square \\
 & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & R_M & \square & \square & \cdots & \square
\end{array}
$$

$\rightarrow$ Ex. 3×2 matrix $\Rightarrow$ Player 1 has 3 choices, Player 2 has 2 choices.

Dominance:

Ex.

$$
\begin{array}{ccccc}
 & \alpha & & \beta & \\
A & 5, 3 & \leftarrow & 7, 1 & \\
 & \uparrow & & \uparrow & \\
B & 3, 5 & \leftarrow & 6, 0 &
\end{array}
$$

arrows show preferences

- A dominates B, B is dominated by A.
- α dominates β, β is dominated by α.

strict dominance: X strictly dominates Y (Y strictly dominated by X) if X is always better than Y.

→ rational players will never choose a strategy that is strictly dominated

- strict vs. weak dominance

weak dominance: X weakly dominates Y (Y weakly dominated by X) if X always at least as Y and sometimes better.

→ strict dominance more useful concept than weak.

Iterated Elimination of Strictly Dominated Strategies (IESDS):
- if X strictly dominates Y then Y never preferred so can be eliminated
- simplified matrix may also have strict dominance
- repeat elimination to "solve" (reveal important outcomes or subgames)

Ex.

$$\begin{array}{c|cc} & \alpha\ \text{NE} & \beta \\ A & \boxed{5,3} & \cancel{7,1} \\ B & \cancel{3,5} & \cancel{6,0} \end{array} \quad \text{or} \quad \begin{array}{c|cc} & \alpha\ \text{NE} & \beta \\ A & \boxed{5,3} & \cancel{7,1} \\ B & \cancel{3,5} & \cancel{6,0} \end{array}$$

→ IESDS will never eliminate NE

IEWDS (IE Weakly DS)
- also allow elimination of weakly dominated strategies
- may eliminate NE
- final simplified game can depend on order strategies eliminated

→ IEWDS not as useful as IESDS

Pareto optima:
- NE not always outcomes that players like (outcomes that players are trapped in)
- alternative solution concept = Pareto optimality

Pareto improvement: change of strategy that raises at least one player's payoff without lowering anyone else's

Pareto optimum (PO): outcome without further Pareto improvements
→ outcome where every change of strategies results in lower payoff for at least one player

unlike NE
- takes into account payoffs of all players
- does <u>not</u> restrict changes to unilateral ones

→ PO can give very different "solutions" than NE

Ex.

	L	C	R
U	73, 25	57, 42	66, 32
M	80, 26	35, 12	32, 54
D	28, 27	63, 31	54, 29

- Find NE and PO
- Simplify by IESDS

Sol'n

- NE via preference arrows

	L	C	R
U	73, 25 →	57, 42	← 66, 32
M	80, 26	35, 12 →	32, 54
D	28, 27 →	$\boxed{63, 31}$ NE	← 54, 29

- NE doesn't have arrows pointing out

⟶ (D, C) or (63, 31) is only NE

- PO by removing outcomes that allow Pareto improvements

	L	C	R
U	~~73, 25~~	$\boxed{57, 42}$ PO	$\boxed{66, 32}$ PO
M	$\boxed{80, 26}$ PO	~~35, 12~~	$\boxed{32, 54}$ PO
D	~~28, 27~~	~~63, 31~~	~~54, 29~~

- Simplify by IESDS:

	L	C	R
U	~~73, 28~~	57, 42	66, 32
M	~~80, 26~~	~~35, 12~~	~~32, 54~~
D	~~28, 27~~	63, 31	54, 29

1. L dominated by R.
2. M " U or D.

	C	R
U	~~57, 42~~	~~66, 32~~
D	$\boxed{63, 31}$	~~54, 29~~

3. R " C.
4. U " D.

→ (D, C) or (63, 31) only outcome after IESDS.
Also NE!

Summary:
- M × N payoff matrices
- strict vs. weak dominance
- IESDS.
- Pareto improvements and Pareto optima (PO).