

UBC ISCI 3444 Game Theory

Dominance and Pareto Optimality

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Outline:

- payoff matrices
- dominance
- strict vs. weak dominance
- iterated elimination of strictly dominated strats.
- Pareto improvement
- Pareto optimality

Payoff matrices:

- $M \times N$ payoff matrix: 2 players, row player has M strategies, column player has N

		Column			
		C_1	C_2	$\dots C_N$	
Row	R_1	□	□	...	□
	R_2	□	□	...	□
	:	:	:	⋮	:
	R_M	□	□	...	□

→ Ex. 3×2 matrix \Rightarrow Player 1 has 3 choices,
Player 2 has 2 choices.

Dominance:

Ex.

		α	β
A	$5, 3 \leftarrow ? , 1$		
B	$3, 5 \leftarrow 6, 0$		

arrows show
preferences

- A dominates B, B is dominated by A.
- α dominates β , β is dominated by α .

strict dominance: X strictly dominates Y (Y strictly dominated by X) if X is always better than Y.

→ rational players will never choose a strategy that is strictly dominated

- strict vs. weak dominance

weak dominance: X weakly dominates Y (Y weakly dominated by X) if X always at least as good as Y and sometimes better.

→ strict dominance more useful concept than weak.

Iterated Elimination of Strictly Dominated Strategies (IESDS):

- if X strictly dominates Y then Y never preferred so can be eliminated
- simplified matrix may also have strict dominance
- repeat elimination to "solve" (reveal important outcomes or subgames)

Ex.

A	α	NE	B
	5, 3		X

or

B	3, 5	6, 0
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A	α	NE	B
	5, 3		X

B	3, 5	6, 0
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→ IESDS will never eliminate NE

IEWDS (IE Weakly DS)

- also allow elimination of weakly dominated strategies
- may eliminate NE
- final simplified game can depend on order strategies eliminated

→ IEWDS not as useful as IESDS

Pareto optima:

- NE not always outcomes that players like (outcomes that players are trapped in)
- alternative solution concept = Pareto optimality

Pareto improvement: change of strategy that raises at least one player's payoff without lowering anyone else's

Pareto optimum (PO): outcome without further Pareto improvements

→ outcome where every change of strategies results in lower payoff for at least one player

unlike
NE

- takes into account payoffs of all players
- does not restrict changes to unilateral ones

→ PO can give very different "solutions" than NE

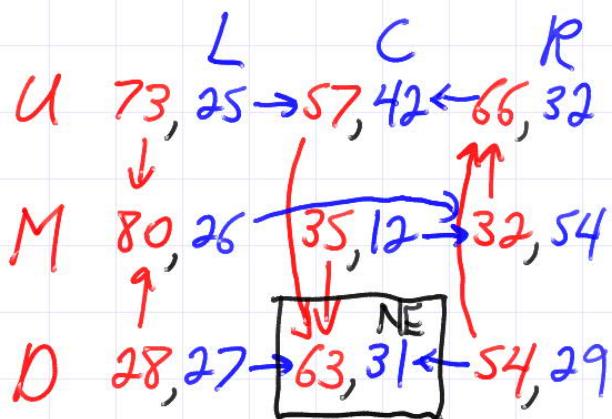
Ex.

	L	C	R
U	73, 25	57, 42	66, 32
M	80, 26	35, 12	32, 54
D	28, 27	63, 31	54, 29

- Find NE and PO
- Simplify by IESDS

Sol'n

- NE via preference arrows



- NE doesn't have arrows pointing out

→ (D, C) or (63, 31) is only NE

- PO by removing outcomes that allow Pareto improvements

	L	C	R
U	73, 25	57, 42^{PO}	66, 32^{PO}
M	80, 26^{PO}	35, 12	32, 54^{PO}
D	28, 27	63, 31	54, 29

- Simplify by IESDS:

	L	C	R
U	73, 28	57, 42	66, 32
M	80, 26	35, 12	32, 54
D	28, 27	63, 31	54, 29

1. L dominated by R.
2. M " C or D.

	C	R
U	57, 42	66, 32
D	63, 31	54, 29

3. R "
4. U "

C:
D:

→ (D C) or (63 31) only outcome after IESDS.
Also NE!

Summary : • $M \times N$ payoff matrices
• strict vs. weak dominance
• IESDS.
• Pareto improvements and Pareto optima (PO).