UBC ISCI 344 Game Theory
Sotto vs. Blotto and mixed Nash equilibria
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Outline:
- Sotto vs. Blotto game
- mixed strategies
- mixed Nash equilibria (NE)

Sotto vs. Blotto:

- Colonel Blotto holds 2 valuable cities
- Colonel Sotto wants to take them
- City 1 twice as valuable as City 2
- each colonel commands one indivisible army
  - → Sotto can attack City 1 <u>or</u> City 2 (not both)
    Blotto " defend "
- if Sotto and Blotto choose same city then Blotto successfully defends → no change
- if they choose different cities then Blotto loses city to Sotto

		Blotto Defend 1	Blotto Defend 2
Sotto	Attack 1	0,0	10,-10
Sotto	Attack 2	5,-5	0,0

- dominance? No.
- Pareto optima? — all of the outcomes
- NE?

		Blotto	
		Defend 1	Defend 2
Sotto	Attack 1	0, 0	10, -10
	Attack 2	5, -5	0, 0

- no NE found

Nash's existence theorem: every finite game has at least one NE

→ so where is the NE?

Mixed strategies:

- pure strategies: definite choices available
  (eg. "Up", "Down", "Offer high", "Accept all", "Attack 1", "Defend 2")
- mixed strategies: action chosen randomly from two or more pure strategies, with non-zero probabilities

→ Ex, Sotto could flip a coin to decide which city to attack

- define: p = probability Sotto attacks City 1
  q = " Blotto defends "

→ p and q characterize mixed strategies

Expected utilities:

- what is "outcome" when facing a player with mixed strategy?
  → instead of known outcomes we have <u>expected</u> outcomes

- what is utility/payoff of an expected outcome?

  → <u>expected utility</u>: sum of utilities weighted by probabilities

		Blotto: Defend 1 (q)	Blotto: Defend 2 ($1-q$)
Sotto	Attack 1 (p)	0,0	10,−10
Sotto	Attack 2 ($1-p$)	5,−5	0,0

Sotto: $U_S(\text{Attack 1}) = q\cdot(0) + (1-q)\cdot(10) = 10 - 10q$
$U_S(\text{Attack 2}) = q\cdot(5) + (1-q)\cdot(0) = 5q$

Blotto: $U_B(\text{Defend 1}) = p\cdot(0) + (1-p)(-5) = 5p - 5$
$U_B(\text{Defend 2}) = p\cdot(-10) + (1-p)\cdot(0) = -10p$

- what probabilities (p and q) should Sotto and Blotto choose?
- recall NE: no player has an incentive to unilaterally change strategy

→ mixed NE: assume p^*, q^* are mixed strategies where neither player has an incentive to switch; ie. expected utility for player 1 equal or lower for any $p \neq p^*$ when $q = q^*$, and vice versa. Then p^*, q^* is mixed NE

- if expected utilities for Attack 1 and Attack 2 same then Sotto indifferent, has no incentive to switch

  → regardless of choice of p, expected utility remains the same

- Blotto can enforce Sotto's indifference by choosing q such that

$$U_S(\text{Attack } 1) = U_S(\text{Attack } 2)$$
$$10 - 10q = 5q$$
$$10 = 15q \Rightarrow q = 2/3$$

- same argument applies to Blotto. Can be made indifferent by Sotto choosing p:

$$U_B(\text{Defend } 1) = U_B(\text{Defend } 2)$$
$$5p - 5 = -10p$$
$$15p = 5 \Rightarrow p = 1/3$$

- if Sotto and Blotto choose city 1 with probabilities $p = 1/3$ and $q = 2/3$, respectively, then neither has incentive to switch → condition for NE!

→ $(p^*, q^*) = (1/3, 2/3)$ is called mixed NE

→ in contrast, previously discussed NE (composed of pure strategies) are called pure NE

Summary:
- Sotto vs. Blotto game
- Nash's existence theorem
- pure vs. mixed strategies
- expected utility
- pure vs. mixed NE