UBC ISCI 344 Game Theory
Evolutionary Game Theory

Rik Blok and Christoph Hauert

Outline:
- simple population model
- evolutionary model
- solve dynamics — equilibria and stability
- evolutionary game theoretic model
- replicator equation

Simple population model:

- population of replicating individuals, A
- one species, asexual, haploid $\longrightarrow$ clones
- population size can change over time, $a(t)$
- depends on fitness parameter, f_A
- exponential growth or decline

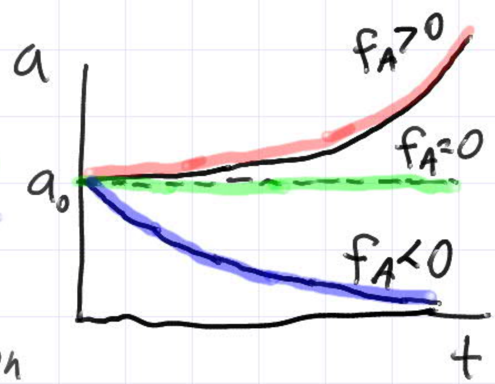
$$\frac{da}{dt} = f_A a \longrightarrow a(t) = a_0 e^{f_A t}$$

- check solution: $\frac{da}{dt} = \frac{d}{dt}\left(a_0 e^{f_A t}\right) = a_0 f_A e^{f_A t} = f_A a$ ✓
- what happens in long run?

  $f_A > 0$: explosion, $a(t) \to \infty$

  $f_A = 0$: constant, $a(t) = a_0$

  $f_A < 0$: extinction, $a(t) \to 0$



- ecological model, just about population size $\longrightarrow$ not evolutionary

Evolutionary model:

- evolution about changes in trait frequency
- need second "type" $\rightarrow$ single gene, 2 alleles/types: A or B
- track numbers of both types: $a(t)$ and $b(t)$
- selection $\rightarrow$ each type has fitness: f_A vs. f_B

$$\frac{da}{dt} = f_A a \;,\quad \frac{db}{dt} = f_B b \;\longrightarrow \text{exponential}$$

- will track frequency (or proportion) of each type

$$\rightarrow x = \frac{a}{a+b} = \text{frequency of A's}$$

$$1-x = \frac{b}{a+b} = \quad '' \quad \text{B's}$$

- quotient rule to find dx/dt

$$\frac{dx}{dt} = \frac{\frac{da}{dt}(a+b) - a\left(\frac{da}{dt} + \frac{db}{dt}\right)}{(a+b)^2} = \frac{1}{(a+b)^2}\left(b\frac{da}{dt} - a\frac{db}{dt}\right)$$

$$= \frac{1}{(a+b)^2}\left(b f_A a - a f_B b\right) = \frac{a}{a+b}\,\frac{b}{a+b}\left(f_A - f_B\right)$$

$$= x(1-x)(f_A - f_B)$$

Solve dynamics — equilibria:

- interested in long-term trends (what eventually happens)
- first look at equilibria — special values where x doesn't change

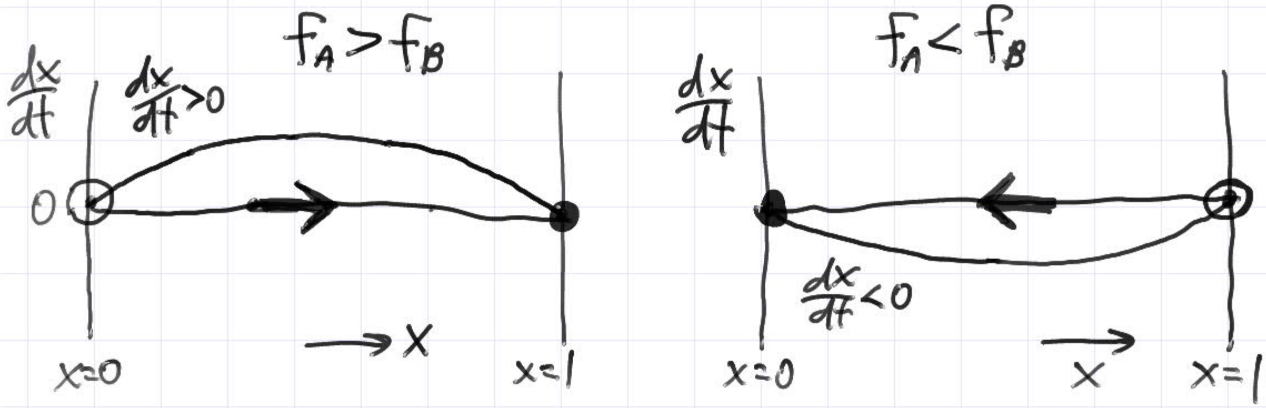
$$dx/dt = 0 \longrightarrow x=0 \text{ or } x=1$$

$\rightarrow$ if A's lost ($x=0$) or B's lost ($x=1$) then population all one type, no further evolution

$\rightarrow$ $f_A = f_B$ also gives $dx/dt = 0$ (for <u>all</u> x) but unlikely to occur if f_A and f_B are arbitrary constants

Solve dynamics — stability:

- interested in what happens for all $0 \le x \le 1$



→ ● = stable equilibrium, <u>will</u> return if disturbed
   O = unstable equilibrium, won't "

Evolutionary game theoretic model:

- where are the games?
  - → hidden in fitness of A & B: f_A and f_B
  - → game result if fitness depends on interactions with other A and B types

	A	B
A	a	b
B	c	d

symmetric game

→ f_A = average fitness/payoff of A's in population
   f_B = " B's "

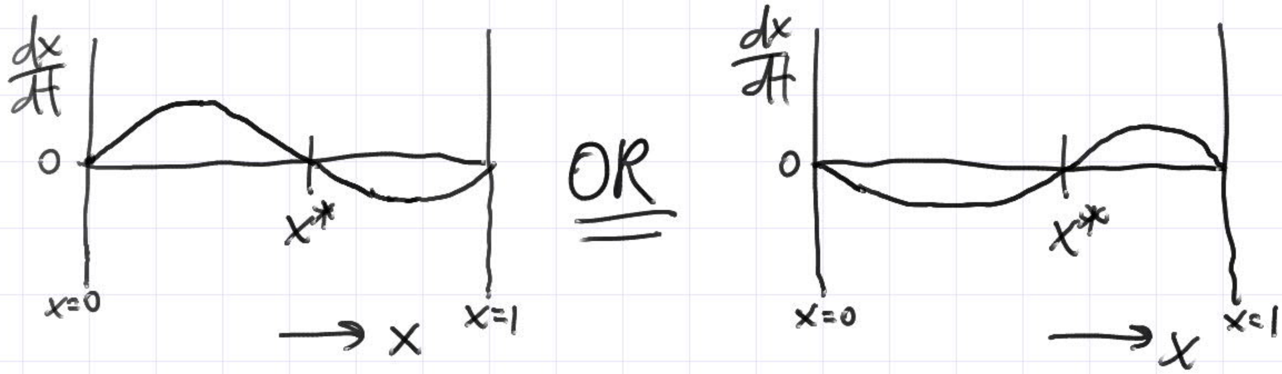
→ like "expected utility" but utility is fitness

$$f_A = ax + b(1-x)$$
$$f_B = cx + d(1-x)$$

- recall eq.: $x=0$, $x=1$, or $f_A = f_B$
  - → f_A and f_B no longer constant so may be a third x^*

$$f_A = f_B \longrightarrow x^* = \frac{d-b}{a-c+d-b}$$

  - → only meaningful if $0 < x^* < 1$ (interior equilibrium)
  - → then two more possibilities



Replicator equation:

- derived $\frac{dx}{dt} = x(1-x)(f_A - f_B)$ where $f_A = xa + (1-x)b$
  $f_B = xc + (1-x)d$
- consider $\bar{f}$ = average fitness in whole population
  $= x f_A + (1-x) f_B$
  - → $\frac{dx}{dt} = x(f_A - \bar{f})$ replicator equation
  - → frequency of A's increases if fitness of A above average
    " B's " B "
- replicator equation will be basis for our evolutionary game theory

Closing thoughts:

- how does individual (eg. bacterium) choose between A & B?
  - → it doesn't. Is genetically determined to be A or B
    Selection determines whether its descendents thrive

- why symmetric game?
  - → hint: what would an asymmetric game mean biologically?

Summary:
- built simple population model
- added evolution
- solved dynamics (equilibria and stability)
- added game theory
- replicator equation