UBC ISCI 344 Game Theory
An Asymmetric Evolutionary Game

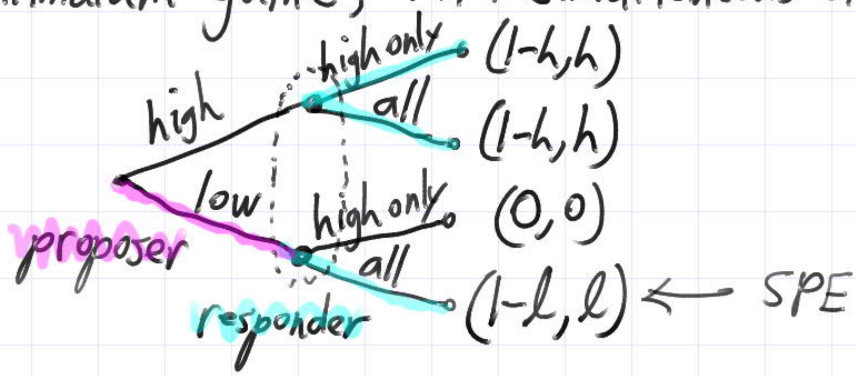
Rik Blok and Christoph Hauert

Outline:
- an asymmetric game
- economic solution concepts (NE, PO)
- evolution: how to handle asymmetry?
  - → separate populations
  - → randomly assign roles

An asymmetric game:

- we've looked at some asymmetric games already
  - → Ultimatum game
  - → Matching pennies
  - → Sotto vs. Blotto
  - → Battle of the Sexes

- Ultimatum game, with simultaneous decisions



$0 < \ell < h < 1$

- responder ignorant of proposer's strategy → simultaneous game

- payoff matrix

$$\begin{array}{cc|ccc} & & (q) & \text{Responder} & (1-q) \\ & & \text{high only} & & \text{all} \\ \hline & (p)\ \text{high} & 1-h, h\ \text{PO}\ \boxed{\text{WNE}} & \Longleftrightarrow & 1-h, h\ \text{PO} \\ \text{Proposer} & & \uparrow & & \downarrow \\ & (1-p)\ \text{low} & 0,0 & \longrightarrow & 1-\ell, \ell\ \text{PO}\ \boxed{\text{SNE}} \end{array}$$

- preference arrows
- NE, PO

- mixed NE? Use "endpoints shortcut"
- proposer: choose p^* to make responder indifferent

$$U_{res}(\text{high only}) = h p^* + (0)(1-p^*) = h p^*$$
$$U_{res}(\text{all}) = h p^* + \ell(1-p^*)$$
$$U_{res}(\text{high only}) = U_{res}(\text{all})$$
$$h p^* = h p^* + \ell(1-p^*)$$
$$\rightarrow p^* = 1$$

- responder: choose q^* to make proposer indifferent

$$U_{pro}(\text{high}) = (1-h) q^* + (1-h)(1-q^*) = 1-h$$
$$U_{pro}(\text{low}) = (0) q^* + (1-\ell)(1-q^*) = (1-\ell)(1-q^*)$$
$$U_{pro}(\text{high}) = U_{pro}(\text{low})$$
$$1-h = (1-\ell)(1-q^*)$$
$$\rightarrow q^* = 1 - \frac{1-h}{1-\ell} = \frac{h-\ell}{1-\ell} \qquad 0 \le q^* \le 1 \quad ✓$$

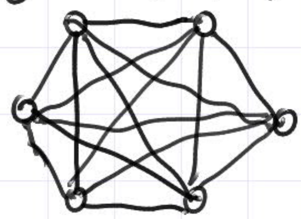
- solutions: 2 pure NEs: weak @ (high, high only)
  strict @ (low, all)
  1 mixed NE: $(p^*, q^*) = \left(1, \frac{h-\ell}{1-\ell}\right)$

Evolution: how to handle asymmetry?

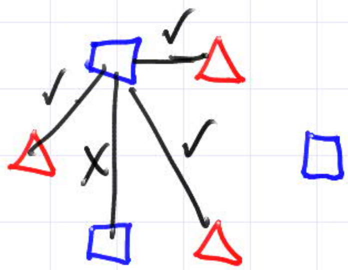
- replicator eq'n: $\frac{dx}{dt} = x(1-x)(f_A - f_B)$
  $= x(f_A - \bar{f})$

  where A & B are strategies/types and x = frequency of A's

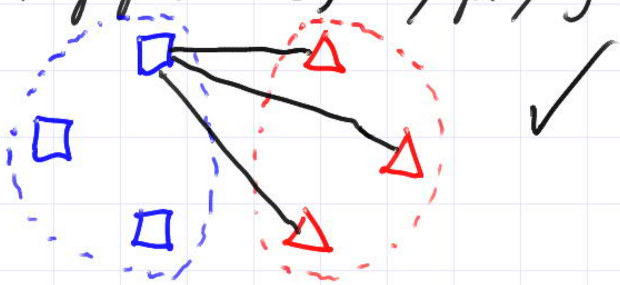
- f_A, f_B = fitness, average payoff of sampling many pairs to play game, eg. every player vs. every other player



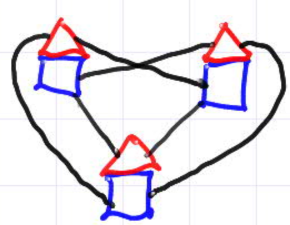
- what if separate roles (asymmetry)? Eg. proposer = □
  responder = △



- 2 solutions:
  → separate populations, only play games across populations



  → randomly assign role, strategies for both roles □ + △ = ⌂



Separate populations:

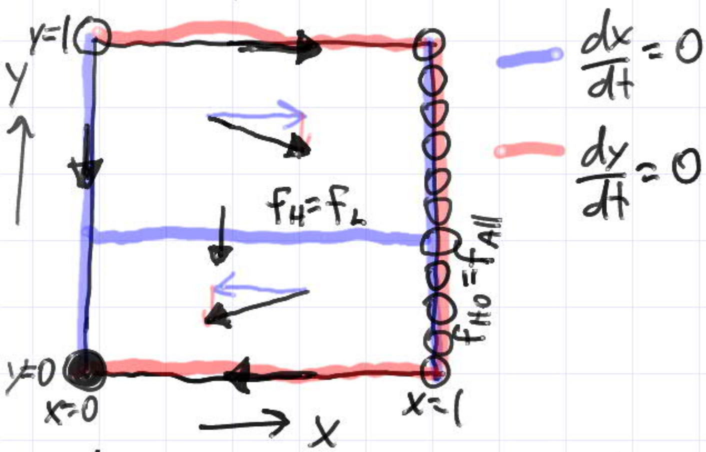
- role fixed, only play against other population
- courtship/mating between males and females
- apply to Ultimatum game

- proposer pop'n: "high" or "low" types, x = frequency of "high"
- responder pop'n: "high only" or "all", y = frequency of "high only"
- 2 replicator equations:

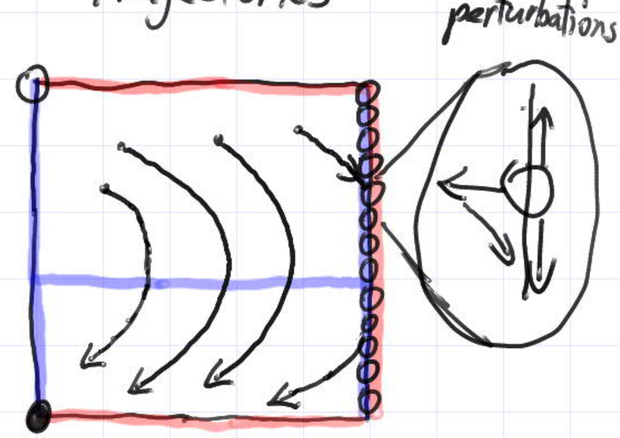
$$\boxed{\frac{dx}{dt} = x(1-x)(f_H - f_L)}, \quad \boxed{\frac{dy}{dt} = y(1-y)(f_{HO} - f_{All})}$$

- payoffs depend on <u>other</u> population
  - → proposer: $f_H = y(1-h) + (1-y)(1-h) = 1-h$ "High"
    $f_L = y(0) + (1-y)(1-\ell) = (1-y)(1-\ell)$ "Low"
  - → responder: $f_{HO} = x(h) + (1-x)(0) = xh$ "High only"
    $f_{All} = x(h) + (1-x)(\ell) = \ell + x(h-\ell)$ "All"
- can draw 2-d "phase plane" to find equilibria and stability

Phase plane



Trajectories



- dynamics:
  - when many high only responders (y large) then increase in freq. of high proposers (x increases)
  - meanwhile "all" responders increases (y decreases)
  - eventually low proposer can thrive and invade
  - stable eq. $(x,y) = (0,0)$ → only "low" offers and accept "all".

Randomly assign roles:

- every player has strategies for both roles
  - → imagine 2 genes with 2 alleles each
  - → 4 types: (proposer, responder) = (H,HO), (H,All), (L,HO), (L,All)
  - → all 4 types play against each other
  - → toss a coin to assign roles
- eg. sharing a lucky find
- Ultimatum game

"Types"	(Freq)		H,HO (x_1)	H,All (x_2)	L,HO (x_3)	L,All (x_4)
①	(x_1)	H,HO	$\frac{1}{2}(1-h+h)=\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}(1-h)$	$\frac{1}{2}(1-h)$
②	(x_2)	H,All	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}(1-h+l)$	$\frac{1}{2}(1-h+l)$
③	(x_3)	L,HO	$\frac{1}{2}h$	$\frac{1}{2}(1-l+h)$	0	$\frac{1}{2}(1-l)$
④	(x_4)	L,All	$\frac{1}{2}h$	$\frac{1}{2}(1-l+h)$	$\frac{1}{2}l$	$\frac{1}{2}$

symmetric game!

$x_1+x_2+x_3+x_4=1$

- replicator equations for 4 types:

$$f_1 = x_1\left(\tfrac{1}{2}\right) + x_2\left(\tfrac{1}{2}\right) + x_3\tfrac{1}{2}(1-h) + x_4\tfrac{1}{2}(1-h)$$

$$\vdots$$

$$f_4 =$$

$$\bar{f} = f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4$$

$$\frac{dx_1}{dt} = x_1(f_1 - \bar{f})$$

$$\vdots$$

$$\frac{dx_4}{dt} = x_4(f_4 - \bar{f})$$

- analysis can be difficult → ask us if interested

Summary:
- asymmetric games
- simultaneous Ultimatum game
  → PO, pure & mixed NE
- evolution: how to handle asymmetry?
- 1) separate populations
  → phase plane, equilibria, stability
- 2) randomly assign roles
  → 4x4 symmetric game
  → may require help to solve