ISCI 344 Game Theory
Multiplayer games
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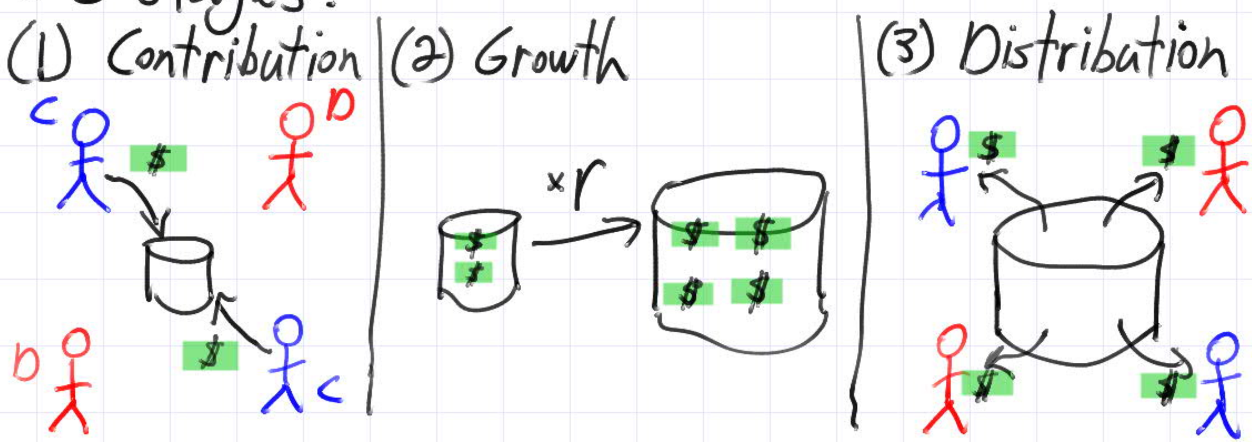
Outline:
- Public goods game
- economic solution
  → conflict of interest/dilemma
- evolutionary solution
- the problem of cooperation

Public goods game:
- multiplayer: group interactions, group size N
- two strategies:
  → Cooperators (C) pay a cost c to contribute to a common good
  → Defectors (D) contribute nothing
- 3 stages:



→ example: $N=4$, $r=2$
- investment is <u>public</u> good, shared equally among <u>all</u> players

Economic solution:

- given k other cooperators in group, what "should" I do?
- payoffs:
  → If I defect, $P_D(k) = kcr/N$
  → If I cooperate, $P_C(k) = (k+1)cr/N - c$
  $$= P_D(k) - \left(1 - \frac{r}{N}\right)c$$
- rationality: choose higher payoff
  → if $r < N$ then $P_D(k) > P_C(k)$ → always defect
  → if $r > N$ then $P_C(k) > P_D(k)$ → always cooperate
- what is NE? Can't write down payoff matrix
- but one strategy dominates for any k, so other strategy can be eliminated. Only one rational outcome, must be NE
  → if $r<N$ then everyone playing D is NE
  → if $r>N$ " C is NE
- what are payoffs if everybody plays C or D?
  $$P_{AllC} = P_C(N-1) = Ncr/N - c = c(r-1)$$
  $$P_{AllD} = P_D(0) = (0)cr/N = 0$$
  → if $r>1$ then AllC is mutually preferred
  → social dilemma for $1<r<N$ because NE is not mutually preferred

Evolutionary solution:

- population with two types: C, D
- x = frequency of C-types
- randomly sample groups of N, payoffs represent fitness
- to determine fitness of a C or D, need to know how many <u>others</u> in group are cooperators, $\bar{k}$, on average

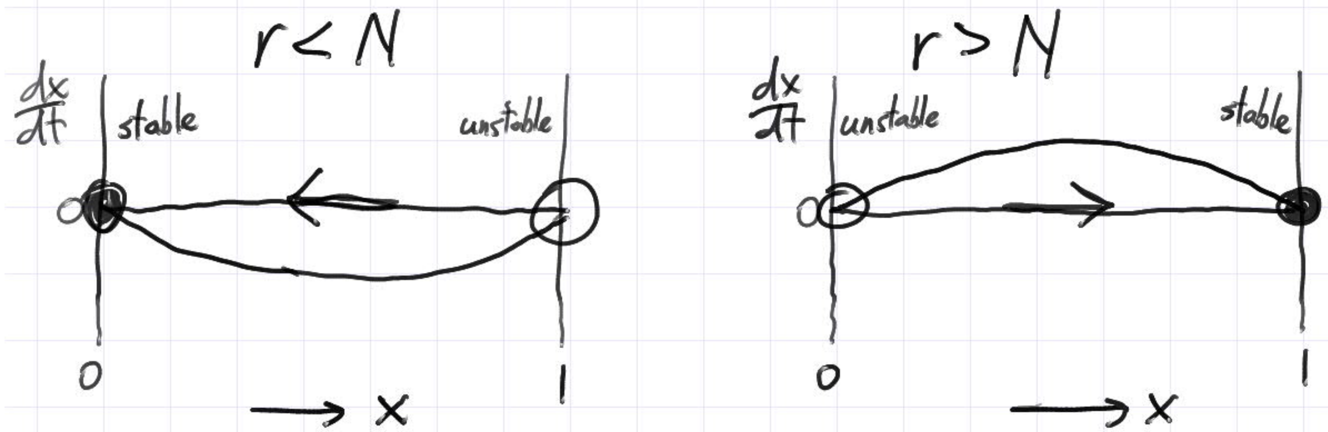
$\bar{k} = (N-1)x$

$f_D = P_D(\bar{k}) = x(N-1)cr/N$

$f_C = P_C(\bar{k}) = (x(N-1)+1)cr/N - c = f_D - \left(1-\frac{r}{N}\right)c$

- replicator equation

$$\frac{dx}{dt} = x(1-x)(f_C - f_D) = x(1-x)\underbrace{\left(-\left(1-\frac{r}{N}\right)c\right)}_{\substack{<0 \text{ if } r<N \\ >0 \text{ if } r>N}}$$



cooperators go extinct   defectors go extinct

Aside: Simpson's paradox

- in each group, defectors do better than cooperators because $P_D(k) > P_C(k-1)$ even when $r>N$. But cooperators invade defectors when $r>N$. What's going on?

→ Simpson's paradox – cooperators more likely to be in groups with more cooperators. On average, cooperators see $\bar{k}+1$ (including themselves) but defectors see $\bar{k}$.

Problem of cooperation:

- consider fitness of equilibria

$$\left.\begin{array}{l} x=1:\ f_C(x=1) = c(r-1) \\ x=0:\ f_D(x=0) = 0 \end{array}\right\} \text{ if } r>1 \text{ then } f_{AllC} > f_{AllD}$$

- when $1<r<N$ cooperation is lost, even though a population of AllC has a higher fitness than All D.
  - → selection can't support cooperation with our setup
  - → how can cooperation evolve?

Summary:
- public goods game
- economic solution
  - → All D is NE when $r<N$
  - → All C is mutually preferred when $r>1$
  - → social dilemma when $1<r<N$
- evolutionary solution
  - → All D is stable equilibrium when $r<N$
  - → All C would yield higher fitness when $r>1$
  - → "evolutionary dilemma" when $1<r<N$ (problem of cooperation)