ISCI 344 Game Theory
Repeated games
Rik Blok and Christoph Hauert

Outline:
- problem of cooperation
- memory-one strategies
- solutions
- danger of short-term rationality
  - → long-term rationality
  - → shadow of the future

Problem of cooperation:
- how to maintain beneficial cooperation when temptation to defect
- Prisoners Dilemma: pay cost c to give benefit b to other player

	C	D
C	b-c	-c
D	b	0

$b > c > 0$

- repeated game? Play m rounds

Memory-one strategies:
- each round choose C or D, "short-term" strategies
- how to choose? Choice can depend on history
- "long-term" strategies, conditional response to past interactions
- simple case: memory-one strategies only depend on prior round

- examples:
  - → allD: always choose D
  - → allC: " C
  - → TFT: start with C, then copy other player's prior choice
  - → Grim: " , keep playing C until other player plays D once, then play D for rest of game
  - → Pavlov: "win-stay-lose-shift". Start with C, if outcome was good (b or b-c) then keep doing what you did last time, otherwise switch to opposite
  - ⋮

  many more

- can write memory-one strategies as choice in 1st round and 4 choices depending on outcome of prior round

	1st round	Conditioned on prior round			
		C	C	D	D ← my choice last round
		C	D	C	D ← your choice "
allD	D	D	D	D	D
allC	C	C	C	C	C
TFT	C	C	D	C	D
Grim	C	C	D	D	D
Pavlov	C	C	D	D	C
		⋮			

$2^5 = 32$ pure memory-one strategies

Example: TFT vs. allD:

Per round:

	C	D
C	b-c	-c
D	b	0

		Round 1	2	...	m	Total
vs.	TFT	C(-c)	D(0)	...	D(0)	-c
	AllD	D(b)	D(0)		D(0)	b
	TFT	C(b-c)	C(b-c)	...	C(b-c)	m(b-c)
	TFT	C(b-c)	C(b-c)		C(b-c)	
	AllD	D(0)	D(0)	...	D(0)	0
	AllD	D(0)	D(0)		D(0)	

- long game payoff matrix

	TFT	AllD
TFT	m(b-c)	-c
AllD	b	0

- what kind of game? Depends on number of rounds, m
  2 cases:

$m < \frac{b}{(b-c)}$ $\qquad$ $m > b/(b-c)$

	TFT	AllD
TFT	↓	↓
AllD		

(PD)

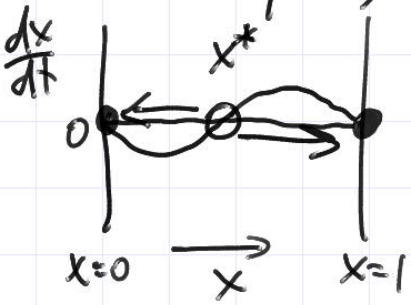
	TFT	AllD
TFT	↑	↓
AllD		

(Stag hunt)

- so PD becomes Stag hunt if repeated for many rounds with long-term strategies TFT and AllD

Solution – evolution:

- let x = frequency of TFT



- all TFT is stable equilibrium and population will evolve there if starting $x > x^*$

Solution – economic:

- Stag hunt has 3 NE: (TFT, TFT) $EU_{TFT} = m(b-c)$
  (allD, allD) $EU_{AllD} = 0$
  mixed

- mixed NE: find q^* that makes row player indifferent

$$EU_{TFT}(q^*) = m(b-c)\,q^* - c(1-q^*)$$
$$EU_{AllD}(q^*) = b\,q^* + 0(1-q^*) = bq^*$$

$$EU_{TFT}(q^*) = EU_{AllD}(q^*) \rightarrow q^* = \frac{c}{(m-1)(b-c)} = p^*$$

(symmetric)

- which NE should players prefer?

$$EU_{TFT} > EU_{AllD}$$

- what about mixed?

$$EU_{mix} = bq^* = \frac{bc}{(m-1)(b-c)} > 0$$

- $EU_{mix} > EU_{AllD}$ so AllD is never preferred NE
- what about TFT vs. mixed NE?

→ recall $m > \frac{b}{b-c}$ so $m-1 > \frac{b}{b-c} - \frac{b-c}{b-c} = \frac{c}{b-c}$

$$m(m-1) > \left(\frac{b}{b-c}\right)\left(\frac{c}{b-c}\right) = \frac{bc}{(b-c)^2}$$

or $m(b-c) > \frac{bc}{(m-1)(b-c)} \rightarrow EU_{TFT} > EU_{mix}$

- so TFT preferred long game strategy
  → mutual TFT always plays C so cooperation saved!

Danger of short-term rationality:
- m rounds of play, what about last round?
- no incentive to cooperate because no future consequences
- consider more sophisticated version of TFT: TFT′ plays TFT but defect in last round
- long game payoff matrix:

	TFT	TFT′
TFT	$(m-1)(b-c)+b-c$	$(m-1)(b-c)-c$
TFT′	$(m-1)(b-c)+b$	$(m-1)(b-c)+0$

- long game becomes PD and TFT′ dominates
- so everybody plays TFT′. What about 2nd last round?
- no incentive to cooperate ...
- ⋮
- argument repeats until we are back to AllD
- short-term rationality leads to choosing AllD over TFT (by backwards induction)
- but we know players rationally prefer TFT over AllD in long-term
  → short-term rationality can lead to long-term irrationality
  → can be long-term rational to forego short-term gains

Shadow of the future:

- how to escape circular argument if players short-term rational?
- end of game unknown, play another round with some probability
- players don't know if this is last round
  → "shadow of the future": fear of future retaliation encourages cooperation in the present

Summary:
- problem of cooperation → repeated game?
- memory-one strategies
- example, TFT vs. AllD
- solutions → repeated game can save cooperation
- danger of short-term rationality
  → forego short-term gains to maximize long-term utility
  → shadow of the future